

An Economic Order Quantity Model for Ameliorating Items with Linear Demand and Shortage

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Abstract— *In this research, we developed an economic order quantity model for ameliorating items where the demand is linear with shortage. Fruits, farm produce, poultry, fish etc, provide good examples for such items. When these items are in the production centre or taken to the sales counter, they incur increase in weight or utility due to growth and at the same time may deteriorate in value due to disease. An economic order quantity (EOQ) model for such items is hereby formulated. The model determines the best cycle length so as to minimize the overall cost. Fruit merchants, poultry managers, etc, could use the model. Numerical examples are given to illustrate the model.*

I. INTRODUCTION

Inventory modeling is an important part of Operation Research, which may be used in variety of problems. To make it applicable in real life situations researchers are engaged in modifying the existing models on different parameters under various circumstances. One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as idle resources that possess economic value by [1]. Usually, it is an important component of the investment portfolio of any production system.

The inventory control system of inventory policy is efficient method that helps in keeping smooth operation of stock held in a store room or building of organization. The deterministic inventory control model is a model under inventory control system which helps in predicting when to place a new order and how much quantity to order for. This helps in avoiding wastage and also helps in monitoring the flow of the stock held by organization so as to reduce scarcity of the good produced by the company.

It is also observed however, that some items when in inventory undergo tremendous increase in quantity or quality or both. Generally, fast growing animals like fishes, poultry, cattle, etc, provide good examples of the items. Some fruit merchants in Nigeria invest huge amount of money in buying large plantations of orange, banana, pineapple, etc and keep such farms for months waiting for the arrival of times of festivities when the demand for these items increase exponentially. Within this period, it is certain that these items (in the farm) undergo increase in quantity and quality. The items that exhibit such properties are referred to as ameliorating items.

The existing literature on inventory seems to ignore or give little attention to the ameliorative nature of inventory [2], for the first time studied an inventory model, the Economic Order Quantity model and Partial Selling Quantity (PSQ) model in connection with ameliorating items under the assumption that the ameliorating time follows the Weibull distribution. Again, [3] developed inventory models for both ameliorating and deteriorating items separately under the Last-In-First-Out (LIFO) and First-In-First-Out (FIFO) issuing policies. Later, [4] developed an EOQ model for ameliorating/deteriorating items under inflation and time discounting. The model studied inventory models with zero-ending inventory for fixed order intervals over a finite planning horizon allowing shortages in all but in the last cycle. They also developed another model with shortages in all cycles taking into account the effects of inflation and time value of money. Later, [5] developed a partial selling inventory model for ameliorating items under profit maximization.

An economic order quantity model for ameliorating inventory where the lead time, the replenishment

time and the demand rate are constants with no shortage of items was studied by [6]. The model obtained an optimum quantity to order while keeping the relevant inventory costs minimum. Again [7] extended their earlier model to allow for linear trend in demand.

In this write up, we extend the work done by [8] who developed a model for items that exhibit amelioration/deterioration with constant demand and shortage by developing an economic order quantity model for ameliorating items with linear demand and shortage. The model determines the optimum cycle length so as to keep the overall costs.

The main aim is to extend the work done by [8] that developed a simple economic order quantity model for both ameliorating and deteriorating items with constant demand and shortage to develop an inventory model for items that exhibit amelioration with linear demand and shortage. The objective of a model is to determine the optimum cycle length so as to minimize the overall costs.

The significance of the study is to use inventory model for items that exhibit amelioration with linear demand and shortage to control the optimum cycle length so as to minimize the overall costs. Generally the customer (suppliers and retailers) need to be guided on the amount to order and when to order

1.1 Objective of the Study

The objective of this study is to develop an economic order quantity (EOQ) inventory model for items that exhibit amelioration under a linear time-dependent demand pattern while allowing shortages. The study aims to determine the optimal cycle length that minimizes the total variable inventory cost by incorporating ordering cost, holding cost, amelioration benefit, and shortage cost into the model. In addition, the research seeks to examine the influence of key system parameters such as demand rate, amelioration rate, holding cost, ordering cost, and shortage cost on the optimal EOQ and total variable cost. Finally, the study provides numerical examples and sensitivity analysis to illustrate the practical applicability of the proposed model in managing real-life inventory systems involving ameliorating items.

1.2 Organization

This article is structured as follows. Section 1 presents the introduction, emphasizing the importance of inventory modeling in real-world applications and clearly outlining the aims and objectives of the study. Section 2 reviews the related literature, focusing on previous studies concerning inventory models for both ameliorating and deteriorating items. Section 3 describes the assumptions and notations of the model, followed by the model formulation, numerical illustrations, and analysis of the results. Section 4 discusses the results in detail and includes the sensitivity analysis. Section 5 presents the conclusions and future scope of the work

II. RELATED WORK

One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as idle resources that possess economic value by [9]. Usually, it is an important component of the investment portfolio of any production system. Keeping an inventory for future sales or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually, the demand rate is decided by the amount of the stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons motivate the customers to buy more.

If the stock is insufficient the customers may prefer some other brands, as shortages will fetch loss to the producers. The shortage or stock out cost is the penalty incurred for being unable to meet the demand when occurs. It has two wings — internal and external shortage. Internal shortage occurs when an order of a group or department within the organization is not filled. External shortages can incur backorder cost, present profit loss and future profit loss. Internal shortages can result in lost production and delay in a completion date. On the other hand, deterioration is an important natural phenomenon and the consequent loss due to decay of items may be quite significant. Mainly when physical goods are stocked for future use, in some items such as medicines, foodstuff, dairy items, volatile liquids,

the process of deterioration is observed. Hence, effect of deterioration is very important in many inventory systems.

Research in this direction began with the work of [10] who considered fashion goods deteriorating at the end of a prescribed storage period. [11] Developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by [12]. In all these models, the demand rate and the deterioration rate were constant and shortages were not allowed.

An interesting subset of inventory modeling is the mathematical modeling of deteriorating items with shortages. The literature related to deteriorating items with shortages is scattered and no comprehensive up-to-date discussion of these models is available. This paper presents a complete survey of the published literature in mathematical modeling of deteriorating items with shortages and proposed future research directions needed in this field, [13] proposed a time-dependent demand pattern by considering it as the combination of linearly time dependent and exponentially time dependent demand in two successive time periods over the entire time horizon and termed as “ramp-type” time dependent demand pattern. Then, inventory models with ramp-type demand rate also studied by [14]. In these papers, the determination of the optimal replenishment policy requires the determination of the time point when the inventory level falls to zero. So the following two cases should be examined:

- This time point occurs before the point where the demand is stabilized.
- This time point occurs after the point where the demand is stabilized.

Almost all of the researchers examine only the first case [15] reconsidered the inventory model of [16] and the models of [8] studied it exploring these two cases, [19] extended the work of [20] by introducing a general ramp-type demand and considering Weibull distribution deterioration rate.

The works done by [21] are some of the models for deteriorating items based on different realistic situations. Deterioration is defined as decay, spoilage,

loss of utility of the product as defined by [22]. Products such as vegetables, fish, medicine, blood, radioactive chemicals have finite shelf life and start to deteriorate once they are produced.

Obsolescence refers to inventories that become obsolete at a certain time, for instance because of rapid changes in technology, or the introduction of a new product by a competitor. If the rate of obsolescence, deterioration or amelioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored [23] considered ameliorating/deteriorating items on an inventory model with time-varying demand pattern. The model of [24] may also be mentioned. [25] noted that ramp-type demand pattern is generally followed by new brand of consumer goods coming in the market. But for fashionable products as well as for seasonal products, the steady demand will never be continued indefinitely. Rather, it would be followed by decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand may be illustrated by three successive time periods that classified time-dependent ramp-type function, viz., first phase the demand increases with time; after that it becomes steady; towards the end in the final phase it decreases and becomes asymptotic. Some consumer goods for which stock-dependent demand pattern can be noticed are subject to deterioration. [26] Investigated a model assuming the demand rate to be linear function of the on-hand inventory incorporating deterioration effect and allowing shortages which are completely backlogged for both infinite and finite time-horizon.

In most of the above-mentioned papers, the demand during stock-out period is totally backlogged. But in real life situations, there are customers who are willing to wait and receive their order at the end of stock-out period due to goodwill of the retailer or for some reasons while others are not. In the last few years considerable attention has been paid to inventory models with partial backlogging. The first work in which customer's impatience functions is proposed seems to be by [27] derived a pricing and ordering policy for a variable rate of deterioration and partially backlogging. The partially backlogging was assumed to be exponential function of waiting time till the next replenishment. [28] Modified this

model taking into consideration the backorder cost and lost sale [29] also developed a deterministic inventory model in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is backlogged which is a function of time. It is assumed that the backlogged units are proportional to waiting time. Thus in this paper, an optimal replenishment schedule is derived under the assumption of waiting time backordering when units in an inventory are subject to constant deterioration. Research on models with partial backlogging for deteriorating items continues with [30].

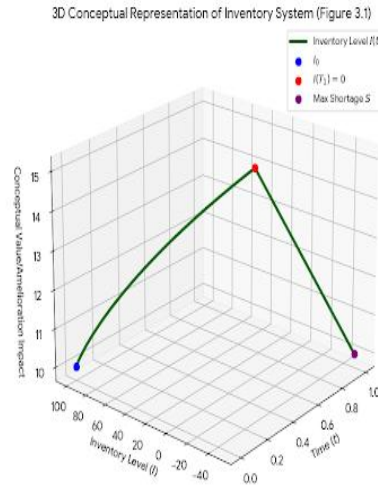
III. ASSUMPTIONS AND NOTATIONS

The propose ameliorating inventory model is developed under the following assumptions and notations;

- The inventory system involves only one single item and one stocking point
- Amelioration occur when the items are effectively in stock
- The cycle length is T
- Initial stock level is I_0
- The inventory carrying cost in a cycle is C_h
- The unit cost of the item is a known constant C
- The replenishment cost is also know C_0 per replenishment
- The demand rate at time t is linear $(a + bt)$ where a is initial demand and b is trend demand
- The total demand is R_T
- The level of positive inventory at any time t is $I(t)$ in the interval $0 \leq t \leq T_1$
- The backorder due to shortage is S in the interval $T_1 \leq t \leq T$
- The unit cost of the items backordered is C_S is known constant
- The backorder cost is $C_B = SC_S$
- Ordering quantity per cycle which enters into the inventory at $t = 0$ is I_0
- Rate of amelioration is β constant
- Inventory holding charge i is known and constant
- The total number of ameliorated amount in $(0, T)$ when considered in terms of value is given by A_T

- The total amount of on hand inventory within the cycle T is I_T

3. The graphical representation of the proposed inventory system is represented in Figure 3.1 below:



3.1 Model Formulation

The propose inventory system is depicted in the graphical representation above. Let $I(T)$ be the on hand inventory at time $t \geq 0$ and let I_0 be the initial inventory at time $t=0$ then at time $t + \Delta t$ the on hand inventory in the interval $(0, T)$ undergoes amelioration and at the same time being depleted by the demand .

Let $I(t)$ be the on hand inventory at time $t \geq 0$, then at time $t + \Delta t$, the on hand inventory in the interval $(0, T)$ is given by:

$$I(t + \Delta t) = I(t) + \beta I(t) \Delta t - (a + bt) \Delta t,$$

Dividing by Δt and taking limit as $\Delta t \rightarrow 0$, we obtain,

$$\frac{dI(t)}{dt} = +\beta I(t) - (a + bt) \quad 0 \leq t \leq T_1,$$

The above equation can be written as;

$$\frac{dI(t)}{dt} - \beta I(t) = -(a + bt) \quad 0 \leq t \leq T_1 \quad (3.1)$$

The solution of equation (3.1) can be obtained using the integrating factor i.e

$$e^{-\beta t} \frac{dI_1(t)}{dt} - \beta I(t)e^{-\beta t} = -(a + bt)e^{-\beta t},$$

$$\frac{d}{dt}(I_1(t)e^{-\beta t}) = -(a + bt)e^{-\beta t},$$

Taking the integral with respect to t of the above equation

$$I(t)e^{-\beta t} = -\int (a + bt)e^{-\beta t} dt,$$

Using the part formula to solve right hand side i.e

$$\int u dv = uv - \int v du,$$

Let $u = (a + bt)$ i.e $du = b dt$,

$$dv = e^{-\beta t} dt \text{ i.e } v = -\frac{1}{\beta} e^{-\beta t},$$

Then,

$$I(t)e^{-\beta t} = -(- (a + bt))e^{-\beta t} + \frac{b}{\beta} \int e^{-\beta t} dt,$$

Again,

$$I(t)e^{-\beta t} = \frac{a + bt}{\beta} e^{-\beta t} + \frac{b}{\beta^2} e^{-\beta t} + K,$$

Thus,

$$I(t) = \frac{a + bt}{\beta} + \frac{b}{\beta^2} + Ke^{\beta t}, \quad 3.2$$

Applying the boundary condition, i.e using $I(0) = I_0$ we have

$$I(0) = \frac{a}{\beta} + \frac{b}{\beta^2} + K,$$

$$K = I_0 - \left(\frac{a}{\beta} + \frac{b}{\beta^2} \right).$$

The value of K the then substituted into equation (3.2) to get,

$$I(t) = \frac{a + bt}{\beta} + \frac{b}{\beta^2} + I_0 e^{-\beta t} - \left(\frac{a}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta t}$$

3.3

Applying the boundary condition $I(T_1) = 0$

$$\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} + I_0 e^{-\beta T_1} - \left(\frac{a}{\beta} + \frac{b}{\beta^2} \right) e^{\beta T_1} = 0,$$

$$I_0 e^{\beta T_1} = \left(\frac{a}{\beta} + \frac{b}{\beta^2} \right) e^{\beta T_1} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right),$$

$$I_0 = \frac{a}{\beta} + \frac{b}{\beta^2} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta T_1}, \quad 3.4$$

The value of I_0 is then substituted into equation (3.3) to get

$$I(t) = \frac{a + bt}{\beta} + \frac{b}{\beta^2} + \left(\frac{a}{\beta} + \frac{b}{\beta^2} \right) e^{\beta t} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta T_1} e^{\beta t},$$

$$I(t) = \frac{a + bt}{\beta} + \frac{b}{\beta^2} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{\beta(t-T_1)},$$

The total amount of on hand inventory within the cycle time T is,

$$I_T = \int_0^{T_1} I(t) dt,$$

$$= \int_0^{T_1} \left(\frac{a + bt}{\beta} + \frac{b}{\beta^2} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{\beta(t-T_1)} \right) dt,$$

$$= \left(\frac{2at + bt^2}{2\beta} + \frac{bt}{\beta^2} - \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{\beta(t-T_1)} \right) \Big|_0^{T_1}$$

Thus,

$$I_T = \frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} +$$

$$\frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1).$$

3.5

The total number of ameliorated amount in $(0, T_1)$ is given by

$$A_T = \beta I_T,$$

$$= \frac{2aT_1 + bT_1^2}{2} + \frac{bT_1}{\beta} +$$

$$\left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1),$$

The backorder due to shortage is calculated as;

$$S = -\int_{T_1}^T (a + bt) dt,$$

$$= -\left(at + \frac{b}{2} t^2 \right) \Big|_{T_1}^T$$

$$S = -a(T - T_1) - \frac{b}{2}(T^2 - T_1^2),$$

Then we have

$$= a(T_1 - T) + \frac{b}{2}(T_1^2 - T^2), \quad 3.6$$

The Inventory Holding cost in a cycle is obtained as;

$$C_h = iCI_T,$$

$$C_h = \left(\frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} + \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right) \quad 3.7$$

The cost of ameliorated amount is given as;

$$C_B = C_S S,$$

$$= C_S \left(a(T_1 - T) + \frac{b}{2}(T_1^2 - T^2) \right), \quad 3.8$$

The Total variable cost is calculated as;

$$TVC = C_0 + C_h + C_B - CA_T$$

$$= C_0 + iCT_T C_S - C\beta I_T,$$

$$= C_0 + C(i - \beta) \left(\frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} + \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right) \quad 3.9$$

$$+ C_S \left(a(T_1 - T) + \frac{b}{2}(T_1^2 - T^2) \right).$$

The total variable cost per unit time is given by;

$$TVC(T) = \frac{C_0}{T} + \frac{C(i - \beta)}{T} \left(\frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} + \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right) + \frac{C_S}{T} \left(a(T_1 - T) + \frac{b}{2}(T_1^2 - T^2) \right) \quad 3.10$$

To obtain the value of T which minimize the total variable cost per unit time, we differentiate (3.10) with respect to T;

$$\frac{d(TVC(T))}{dT} = -\frac{C_0}{T^2} - \frac{C(i - \beta)}{T^2} \left(\frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} + \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right)$$

$$- \frac{C_S}{2T^2} (2aT_1 + bT_1^2 + bT^2) \quad 3.11$$

Equation (3.11) is then equated to zero to obtain the value of T which minimize the total variable cost per unit time,

$$\frac{C_0}{T^2} + \frac{C(i - \beta)}{T^2} \left(\frac{2aT_1 + bT_1^2}{2\beta} + \frac{bT_1}{\beta^2} + \frac{1}{\beta} \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right)$$

$$- \frac{C_S}{2T^2} (2aT_1 + bT_1^2 + bT^2) = 0$$

Multiply through by $2T^2\beta^3$

$$2\beta C_0 + C(i - \beta)$$

$$\left(\beta^2 (2aT_1 + bT_1^2) + 2\beta bT_1 + 2(\beta(a + bT_1) + b)(e^{-\beta T_1} - 1) + \beta^3 C_S (2aT_1 + bT_1^2 + bT^2) \right) = 0.$$

$$EOQ = R_T - A_T,$$

$$= \int_0^{T_1} (a + bt) dt - \beta I_T,$$

$$= \left(at + \frac{bt^2}{2} \right)_0^{T_1} -$$

$$\left(\frac{2aT_1 + bT_1^2}{2} + \frac{bT_1}{\beta} + \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right)$$

$$= aT_1 + \frac{bT_1^2}{2} -$$

$$\left(\frac{2aT_1 + bT_1^2}{2} + \frac{bT_1}{\beta} + \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) (e^{-\beta T_1} - 1) \right)$$

$$= \frac{-bT_1}{\beta} - \left(\frac{aT_1 + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta T_1} + \frac{a + bT_1}{\beta} + \frac{b}{\beta^2},$$

$$= \frac{-bT_1}{\beta} + \frac{a}{\beta} + \frac{bT_1}{\beta} + \frac{b}{\beta^2} - \left(\frac{aT_1 + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta T_1},$$

$$EOQ = \frac{a}{\beta} + \frac{b}{\beta^2} - \left(\frac{a + bT_1}{\beta} + \frac{b}{\beta^2} \right) e^{-\beta T_1} = I_0.$$

3.2 Numerical Illustrations

Table 3.1: Parameter values and the optimal cycle length for the inventory model of items that exhibit amelioration with linear demand and shortage.

S $/No$	b	C	β	C_o	s	I	T_1	T	TVC (T)	E OQ
1 0	60, 000	3 00	0 .7	30, 000	0	0 .3	37 days	20 days	4974 118	8 80
2 5	50, 000	2 00	0 .6	20, 000	5	0 .22	43 days	15 days	2353 437	1 244
3 0	50, 000	2 50	0 .5	10, 000	0	0 .23	29 days	10 days	1570 980	1 273
4 5	45, 000	3 50	0 .66	15, 000	0	0 .21	28 days	25 days	3850 244	4 17
5 2	25, 000	2 50	0 .88	25, 000	0	0 .16	44 days	30 days	4480 668	1 72
6 2	20, 000	2 40	0 .89	25, 000	0	0 .16	47 days	12 days	3603 899	1 84
7 0	24, 000	2 40	0 .57	35, 000	5	0 .18	48 days	35 days	1570 297	9 66
8 5	30, 000	2 40	0 .57	25, 000	0	0 .18	44 days	40 days	1814 282	1 010
9 0	35, 000	2 80	0 .67	30, 000	5	0 .28	39 days	7d ays	2921 064	9 40
1 0	40, 000	2 60	0 .75	31, 000	0	0 .19	24 days	17 days	4627 397	6 71

IV. RESULTS AND DISCUSSION

From the above table it is clear shows that;

- As a increases $TVC(T)$ increases and EOQ increases.
- As b increases $TVC(T)$ increases and EOQ decreases
- As C increases $TVC(T)$ increases and EOQ decreases
- As β increases $TVC(T)$ increases and EOQ decreases
- As C_0 increases $TVC(T)$ increases and EOQ increases
- As i increases $TVC(T)$ increases and EOQ increases
- As C_s increases $TVC(T)$ decreases and EOQ decreases

From the result of the above analysis, it is clear that EOQ and $TVC(T)$ attained depend on a number of parameters, if different values are assigned to one of these parameters and the rest are kept constant, the following observations were noticed.

As expected, the increase in rate of amelioration induced the $TVC(T)$ and EOQ within which the $TVC(T)$ increases the rise in cost of items is accompanied by the fall in EOQ and the rise in ordering cost. The rise in initial demand and trend demand are accompanied by the rise of $TVC(T)$ and EOQ . The high rate of inventory charges however lowers the EOQ and rise the $TVC(T)$. Finally the rise in cost of backorder per item increase $TVC(T)$ and EOQ fall.

4.2 Sensitivity Analysis

Next, we carry out a sensitivity analysis to see the effect of parameter changes on the decision variables. This has been carried out by changing (that is, increasing or decreasing) the parameters by 1%, 5%,

and 25% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are as given in Table2 below.

Parameter	Change in parameter	% change in result		
		T^*	$TVC(T)^*$	EOQ^*
a				
	-25	0	-0.08153	-0.76879
	-5	0	-0.01632	-0.13266
	-1	0	-0.00328	-0.00543
	1	0	0.00324	0.058185
	5	0	0.016281	0.185412
	25	0	0.081488	0.82155
b	-25	11	-18.8	-16.1
	-5	0	-3.693	-4.82
	-1	0	-0.739	-0.942
	1	0	0.738	0.995
	5	0	3.693	4.87
	25	-11	17.875	10.78
C	-25	11	-18.995	10.791
	-5	0	-3.74245	0.0264
	-1	0	-0.774851	0.02638
	1	0	0.74847	0.02637
	5	0	3.742408	0.00267
	25	-11	18.3047	-10.738
β	-25	11	-43.5622	84.24853
	-5	0	-10.6112	10.49053
	-1	0	-2.21204	2.03415
	1	0	2.258291	-1.9414
	5	0	11.76825	-9.4354
	25	-11	71.93201	-46.1875
C_0	-25	-11	-7.0919	-10.7381
	-5	0	-1.2907	0.02637
	-1	0	-0.25818	0.026378
	1	0	0.25813	0.02637
	5	0	1.29075	0.002638
	25	11	6.193843	0.02638
I	-25	-11	15.6179	-10.7381
	-5	0	3.187974	0.026378
	-1	0	0.637578	0.062378
	1	0	-0.63762	0.062378
	5	0	-3.18801	0.062378
	25	11	-16.045	10.79063
C_s	-25	-11	0.165988	0.026378
	-5	0	0.033181	0.026378
	-1	0	0.004723	0.026378
	1	0	-0.00666	0.026378

	5	0	-0.03322	0.026378
	25	11	-0.18578	-10.7381

V. CONCLUSION AND FUTURE SCOPE

5.1 Conclusion

This study presented a mathematical inventory model for ameliorating items under a linear demand structure with allowable shortages. The model effectively describes the dynamic behavior of inventory levels over time, including phases of inventory accumulation, depletion, and shortage. By incorporating deterioration effects and linear demand into the formulation, the model reflects practical inventory scenarios faced by organizations dealing with time-sensitive or perishable products. The analytical framework provides insight into the impact of demand and shortage policies on inventory performance and helps identify critical points such as maximum inventory level and the onset of shortages. The conceptual and three-dimensional graphical representations further enhance the understanding of the system dynamics by illustrating the interaction between time, inventory level, and shortage conditions. The results demonstrate that a well-structured replenishment policy can mitigate the adverse effects of shortages while maintaining service levels. However, the model is developed under deterministic assumptions, including constant demand and deterioration rates, which may limit its direct applicability in highly uncertain environments. Nonetheless, the proposed formulation serves as a useful foundation for inventory decision-making and academic analysis.

5.2 Future Work

Future research may extend the present model by relaxing some of its simplifying assumptions. Incorporating stochastic or nonlinear demand patterns would improve realism and broaden applicability. The inclusion of variable deterioration rates, inflation effects, and time-dependent holding or shortage costs could further enhance the model's practical relevance. Additionally, allowing partial backlogging, trade credit policies, or multiple replenishment cycles may provide deeper insights into real-world inventory systems.

The model can also be expanded to multi-item or multi-warehouse settings and integrated with numerical optimization techniques to determine optimal replenishment and shortage control policies. Such extensions would contribute to developing a more comprehensive and flexible inventory management framework capable of addressing complex operational environments.

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