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Pressure Build Up Analysis with Wellbore Phase Redistribution (Pressure Derivative Approach)

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Abstract- This study presents a new mathematical model which uses type curves, for the analysis of the effects of well bore phase redistribution on pressure build up tests, based on the principle of superposition. Well bore phase redistribution is shown to be a well bore storage effect and is incorporated mathematically into the diffusivity equation. The parameters that affect phase redistribution and gas humping are also documented. The derivative of the dimensionless well bore pressure was included in the type curves and in the data which makes the matching process much easier, and the method used in this study is validated with actual field data. This study permits analysis of many anomalous pressure buildup tests, which previously could not be analyzed quantitatively.

Keywords- Diffusivity equation, Mathematical model, Phase redistribution, Pressure buildup, Type curves

I. INTRODUCTION

Pressure buildup tests and other types of pressure transient analyses (PTA) have been used for many years to evaluate reservoir fluid flow characteristics and well completion efficiency. PTA, or well testing serves as one of the main tools for determining the hydraulic properties of underground porous media and the productivity of wells. It is widely used in several disciplines, including hydrology, petroleum, and geothermal systems. It can be used to ascertain both the static and dynamic properties of reservoirs (Mohammad *et al*., 2025).

To minimize cost and operational risk, many well tests are performed with a surface shut-in with a bottom- hole pressure measurement, rather than bottom- hole shut-in and pressure measurement. However, a surface shut-in includes many factors that can affect the bottom-hole pressure value. The combined effect of these factors is usually referred to as the wellbore storage effect and period (Edward, 2022). Many factors that influence the pressure response in transient flow conditions have been investigated- i.e., the effects of reservoir boundaries and fractures, well bore storage of fluids, and various types of well impairments, skin effect, and completion practices.

The phenomenon of wellbore phase redistribution occurs in the production of the gas condensate reservoir which shows changeable wellbore storage characteristic observed in transient pressure analysis (Yin *et al.,* 2023). The existence of natural fractures is another difficult problem in gas condensate reservoir development, which not only impacts the flow behavior of fluids but also induces a stress-sensitive effect in the formation (Zhou *et al.,*2016).

Theoretically, the pressure buildup in an infinite reservoir should be a linear function of $\ln[(t_p + \Delta t)/\Delta t]$ where t_P is the production time and Δt is the shut-.in time. The effect of stratified producing zones, and irregular geometrical drainage patterns may also contribute unusual characteristics to buildups. However, the effect of still another phenomenon- that of phase redistribution of fluids has been neglected in most buildup studies to date. Fluid movements within the well bore after shut-in occur as a result of after production, packer failure, leaks in the casing or tubing, or buoyancy of the gas phase when both gas and liquid are present in the well. During wellbore pressure buildup tests, fluid segregation and phase redistribution commonly introduce oscillations and anomalies into pressure measurements that complicate PTA interpretation. Fluid segregation and redistribution lead to three distinct pressure response types, with pronounced effects observed at high water cuts, where redistribution is most significant (Corrales *et al.,*2025). Each of these movements can influence the pressure buildup, sometimes sufficiently to negate the use of the data for computing permeability or stabilized buildup pressure.

In analyzing such buildup test data influenced by phase redistribution, the conventional interpretation approach becomes inadequate, because it assumes isotropy. The wellbore phase redistribution (WPR) phenomenon could happen through closing a well with multiphase flow of gas and liquid while performing pressure buildup test. Gravity forces cause the free-gas phase to rise through the liquid column and the liquid phase to move down. The gas bubbles near the bottom of the tubing are at a pressure comparable with formation pressure. As the bubbles grow to the surface after shut-in, they cannot expand if the well bore is in poor communication with the pay zone. Thus, a high- pressure gas column develops at the top of the tubing and a column of liquid develops below that gas. Under these conditions, the total pressure at the sandface is equal to the pressure formed due to gas column plus that created by the liquid column below that gas (Salem,2016).
When analyzing buildup test data influenced by phase redistribution, Salem, 2016 observed that the gas hump occurs in an oil reservoir having permeability ranging from 10 to 100 md or a gas reservoir having permeability varying from 0.1 to 1 md and high positive skin factor. Moreover, he stated that the wells without packers tend to have smaller hump than that with packers. As a result, he plotted a relationship between the sizes of the produced pressure humps in about 75% of their tests (a field south Texas) versus their reservoir productivity index (PI). It is found that that hump size is very high in a low PI reservoir and decreases as the PI increases. In some cases, the hump does not appear in case of low permeability formations. The reason for that may be attributed to higher reservoir pressure than that of fluid column in the tubing. While in case of high permeability reservoirs, the fluid can return to the formation fast to avoid a major increase in bottom hole pressure (BHP) resulting from growing gas bubbles.

An automatic type curve matching method has been presented that matches measured field data with the pressure simulated by Fair's phase redistribution model. It was shown therein with field examples that this technique is especially useful when the conventional semilog analysis methods cannot be applied (Qasem *et al.,* 2002). However, type-curves are advantageous because they may allow test interpretation even when well bore storage distorts most or all of the test data, in that case conventional methods fail.

It is the objective of this study to develop a model for analyzing pressure buildup test data influenced by phase redistribution using type-curves. This model would enable us determine the formation permeability and skin factor as is obtainable using conventional technique. In developing this model, it is assumed that the sand face production rate during the period dominated by well bore storage decreases exponentially from its value at the instant of shut-in to zero at the end of well bore storage. Also, the exponential form was used in representing the phase redistribution pressure function.

One of the main dynamic sources for reservoir characterization is the Pressure transient analysis The main task in well test interpretation is to understand the reservoir properties using the bottom-hole pressure behavior. Unfortunately, one of the main factors that can affect this task is wellbore storage effects, especially in cases that we have a non-ideal one. In pressure build-up test if pressure is below the bubble point or if gas enters into the wellbore, the phase redistribution can happen during the shut-in period. This phenomenon causes pressure anomalies. Besides, due to pressure increase or existence of under- saturated liquid in the wellbore, the gas starts to dissolve, and this leads to pressure decrease (Gholamzadeh *et al.,*2020).

Pressure transient analysis is one of the best ways used for reservoir evaluation. By using different techniques such as pressure derivative plots and type curves, it is possible to determine the reservoir characteristics with reasonable accuracy. Unfortunately, in some cases wellbore effect dynamics can influence the reservoir response and the data can be misinterpreted. A significant amount of the pressure data that are recorded during a well test is affected by wellbore and gauge related effects and do not reflect the reservoir behavior. The problem of wellbore storage occurs, especially during testing of development wells when a surface valve is used to control the flow and shutting of the well (Stewart, 2011; Gholamzadeh *et al.,*2020).

During a pressure buildup test, the increased pressure in the wellbore is relieved through the formation, and the equilibrium between the wellbore and the surrounding formation will be eventually attained. Sometime early during a buildup test, however, the pressure may rise above the formation pressure; and then decrease, causing an anomalous hump on the conventional pressure buildup analysis curves. In less severe cases, the wellbore pressure may not increase sufficiently to reach a maximum buildup pressure (Qasem,2002).

During wellbore pressure buildup tests, fluid segregation and phase redistribution commonly introduce oscillations and anomalies into pressure measurements that complicate Pressure Transient Analysis (PTA) interpretation. Fluid segregation and redistribution lead to three distinct pressure response types, with pronounced effects observed at high water cuts, where redistribution is most significant. Corrective techniques, such as rate normalization and type-curve matching, proved effective in mitigating the impact of redistribution on pressure analysis (Corrales *et al.,*2025),

The association of the pressure hump with phase redistribution indicated that the size of the hump could be correlated with the volume of the gas flowing in the tubing. The phase segregation effects were also considered as a significant unusual behavior to be noticed without proposing any interpretation technique. It was also noted, based on the shape of the log–log plot of the pressure buildup data, that phase redistribution seems to be related to the wellbore storage problem. It was suggested that a multiple-rate analysis technique minimizes the humping effects and reasonable test results were provided (Qasem,2002)

Phase redistribution is one of the most important in gas wells with high liquid production (condensate or water) and volatile oil wells. The longer the wellbore phenomena, the higher the probability to distort reservoir pressure behavior (infinite acting flow regime, linear flow, etc.). In 2016, Adrian *et al*.,2016 mentioned that if the test data is affected by phase redistribution, the conventional pressure transient analysis would be hard to perform, increasing the probability to obtain erroneous values of permeability and skin factor. Nevertheless, in the last years, the use of primary and secondary pressure derivative as a diagnostic plot (Semi log and Log-Log plots), demonstrated to be very useful in low permeability gas wells and weak phase redistribution gas wells (Salem, 2016).

Well-testing is one of the oil and gas industry's most reliable reservoir characterization and description tools (Lopez and Kumar, 2019). It evaluates reservoir performance, characteristics, and near wellbore condition through the measure of fluid dynamics within the reservoir, influenced by fluid properties, reservoir geometry (Stewart,2011) and the presence of heterogeneity (Ganat, 2023). Well-test data consists of the transient pressure measurement records of the reservoir through the wellbore, obtained using specialized sensors designed for this purpose. Analyzing the pressure record from well-tests, known as pressure transient analysis (PTA), involves interpreting these transient pressure changes to derive valuable insights into the reservoir's properties and behavior. The main objectives of PTA are the identification of the underlaying reservoir model and the estimation of reservoir parameters (Hemmati *et al.,*2020).

The pressure derivative analysis method was proposed (Abdalla *et al.,* 2024) which significantly advanced the field of PTA by offering a comprehensive tool for examining all flow regimes encountered during a well- test. However, the accuracy and efficacy of this method can be challenged by two main factors. Firstly, the increasing complexity of theoretical models affects the process of curve fitting, potentially leading to increased errors in the analysis (Dong *et al.,*2022) Traditionally, the analysis of complex models has been prone to higher errors due to the involvement of human factors, which require significant expertise and experience. Secondly, the accuracy of estimating reservoir parameters is significantly compromised by the presence of noise in well-test data. This noise can affect the shape of the derivative plot, complicating the identification of essential reservoir characteristics. When utilizing testing data, the quality of the data collected is a principal issue for geologists, production, and reservoir engineers (Abdalla *et al.*, 2024). Focusing specifically on the impact of noise in well-test data, innovative noise reduction techniques have become crucial for enhancing data analysis accuracy. The presence of noise not only obscures the derivative plot but also makes it challenging to discern critical reservoir characteristics, necessitating sophisticated filtering methods. Recent advancements in digital signal processing and artificial intelligence offer robust tools for noise mitigation, enabling clearer and more precise derivative plots.

This study now develops a mathematical model to analyze pressure buildup test data influenced by well bore phase redistribution using type-curves. The exponential form will be used in representing the phase redistribution pressure function.

II. MODEL DEVELOPMENT

- *ASSUMPTIONS*

1. The principle of superposition holds during the flow and shut-in period.
2. The flow rate is exponential during the period dominated by well bore storage.
3. The E_i function solution to the diffusivity equation and the logarithmic approximation both applies.

A. *MATHEMATICAL FORMULATION*

- MATHEMATICAL ANALYSIS OF PHASE REDISTRIBUTION

For a well where well bore storage occurs, the effects of the storage can be described by Eqn, (1) (Walter and Fair,1981; Edward, 2022) The effect of changing sand-face flow rate on the wellbore pressure also can be obtained from Eqn. (2) (Walter and Fair,1981; Edward, 2022,).

$$\frac{q_{sf}}{q} = 1 - C_D \frac{dP_{WD}}{dt_D} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$\frac{dP_{WD}}{dt_D} = \frac{1}{C_D}\left(1 - \frac{q_{sf}}{q}\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

To describe the effect of well bore phase redistribution, it is noted that not all of the pressure changes in the well bore can be attributed to well bore storage flow rate effects, since some of the pressure change is caused by phase redistribution.

Thus, Eqn. (2) can be modified by adding a term describing the pressure change caused by phase redistribution, as in Eqn. (3), which also can be rearranged to show the sand-face flow rate dependency in Eqn. (4).

$$\frac{dP_{WD}}{dt_D} = \frac{1}{C_D}\left(1 - \frac{q_{sf}}{q}\right) + \frac{dP_{XD}}{dt_D} \quad \dots\dots\dots\dots\dots\dots (3)$$

$$\frac{q_{sf}}{q} = 1 - C_D\left(\frac{dP_{WD}}{dt_D} - \frac{dP_{XD}}{dt_D}\right) \quad \dots\dots\dots\dots\dots\dots (4)$$

Eqn. (4) also can be written in the form of Eqn.(l) by defining a pseudo well bore storage coefficient given in Eqn. (5).

$$C_{eD} = C_D\left(1 - \frac{dP_{xD}}{dt_D} \Big/ \frac{dP_{wD}}{dt_D}\right) \quad \dots\dots\dots\dots\dots\dots\dots (5)$$

It is apparent from Eqn. (5) that well bore phase redistribution is a form of wellbore storage, since when

$$\frac{dP_{XD}}{dt_D} \geq 0, C_{eD} \leq C_D$$

which implies that the effect of. phase redistribution always will cause an apparent lowering of the well bore storage coefficient given by Eqn. (5). In addition, when

$$\frac{dP_{XD}}{dt_D} > \frac{dP_{WD}}{dt_D},$$

The pseudo storage coefficient becomes negative, indicating a reversal in the direction of flow. When this occurs, a pressure buildup test becomes more like a pressure falloff, and the gas hump results (Walter and Fair,1981). By considering the physical process of phase redistribution, certain properties of the phase redistribution function can be inferred. If the gas and liquid phases in the well bore before shut-in behave as homogeneous fluid, the required pressure function must have a value of zero at shut-in (time

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zero). Also, at long times, the phase redistribution must stop so that its derivative with respect to time must approach zero. If it is specified further that no gas enters solution in the liquid phase, then it can be shown that the pressure function must increase monotonically to its maximum value. These conditions are described by Eqn. (6)

$$\lim_{t \to 0} P_X = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6a)$$

$$\lim_{t \to \infty} P_X = C_x \dots (6b)$$

$$\lim_{t \to \infty} \frac{dP_X}{dt_D} = O \dots\dots\dots\dots\dots\dots\dots\dots\dots .. (6c)$$

It is expected that the phase redistribution pressure initially would rise quickly and then slowly approach its maximum value C_X. This observation leads to the following functional representation.

$$P_X = C_X\left(1 - e^{-t/\lambda}\right) \dots\dots\dots\dots\dots\dots .. (7)$$

The parameter C_X. in Eqn. (7) represents the maximum phase redistribution pressure change and **λ** represents the time .at which about 63% of the total change has occurred. An estimate of C_X. can be obtained from Eqn. (3).

$$C_X = \frac{(P_{gef} - P_{whf})}{ln(\frac{P_{gef}}{P_{whf}})} \dots\dots\dots\dots\dots\dots .. (8)$$

So as to keep the dimensionless quantities consistent, the dimensionless phase redistribution pressure function is defined in Eqn. (9)

$$P_{XD} = C_{XD}(1 - e^{t_D}/\lambda_D) \dots\dots\dots .(9)$$

III. DEVELOPMENT OF DIFFUSIVITY EQUATIONS FOR FLOW IN POROUS MEDIA ASSUMPTIONS

1. A 3-Dimensional, radial flow of slightly compressible liquids.
2. An isotropic system.
3. Gravitational forces, negligible.
4. Porosity and viscosity, constant.

In developing these equations, three equations namely, continuity equations (mass balances); appropriate equations of state and Darcy's law will be combined.

B. Continuity Equation for Three-Dimensional Flow:

In developing the continuity equations, a mass balance will be applied to a small element of porous material. The balan>ce has the following form (Lee, 1982).

(Rate of mass flow into element)- (Rate of mass flow out of element) = (rate of accumulation of mass within element). It has dimensions Δx, Δy, and Δz in the x. y, and z-coordinate system. For convenience, the coordinate system is oriented such that gravitational forces are in the (-) z direction. The components of the volumetric rate of flow per unit cross-sectional area (ft/hr) will be denoted by
$U_x, U_y,\ and\ U_z$

The element is shown in Fig. A:

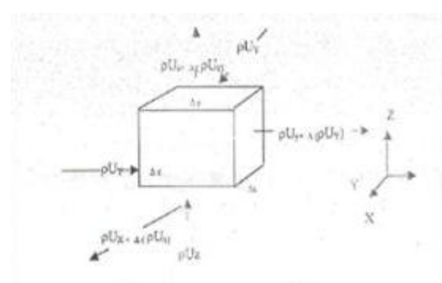

Fig. A- Element of porous medium used for mass balance

The rate at which mass enters the element in the x direction is $\rho U_x \Delta y \Delta z (lbm/hr)$;the rate at which the mass leaves the element in the x direction is $[\rho U_x + \Delta(\rho U_x)]\Delta y \Delta z$. Similar expressions describe rate of mass entering and leaving in the x and y directions. The results of adding these expressions are:

(rate of mass flow into element)- (rate of mass flow out of element) =

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$\rho U_x \Delta y \Delta z + \rho U_y \Delta x \Delta z + \rho U_z \Delta x \Delta y$-$[\rho U_x + \Delta(\rho U_x)]\Delta y \Delta z$-$[\rho U_y + \Delta(\rho U_y)]\Delta x \Delta z$-$[\rho U_z + \Delta(\rho U_z)]\Delta x \Delta y$=-$\Delta(\rho U_x)]\Delta y \Delta z$-$\Delta(\rho U_y)]\Delta x \Delta z$-$\Delta(\rho U_z)]\Delta x \Delta y$

To determine the rate at which fluid accumulates within the element, it is noted that the mass within the element (of porosity φ) at a given time is $\rho \Delta x \Delta y \Delta z (Ibm)$. Thus, the rate at which this mass changes over a time interval is:

$$\frac{((\rho\varphi|t+\Delta t) - \rho\varphi|t)\Delta x \Delta y \Delta z}{\Delta t}$$

Where time, t, is in hours accordingly, the mass balance becomes

$$-\Delta(\rho U_x)]\Delta y \Delta z - \Delta(\rho U_y)]\Delta x \Delta z - \Delta(\rho U_z)]\Delta x \Delta y = \frac{((\rho\varphi|t+\Delta t) - \rho\varphi|t)\Delta x \Delta y \Delta z}{\Delta t}$$

By dividing each term by $\Delta x \Delta y \Delta z$ and taking the limit as $\Delta t, \Delta x, \Delta y, \Delta z \to 0$, the result is:

$$\frac{\partial(\rho U_x)}{\partial x} + \frac{\partial(\rho U_y)}{\delta y} + \frac{\delta(\rho U_z)}{\delta z} = \frac{-\partial(\rho\varphi)}{\delta t} \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

C. Continuity Equation for Radial Flow

The continuity equation for radial flow follows from a similar development. If we consider the elemental volume as shown in figure B, then the following mass balance can be written:

$$-\Delta t\{\theta(r+\Delta r)h(\rho U_r) - \theta r h[\rho U_r + \Delta(\rho U_r)]\} = \emptyset \rho h \theta r \Delta r\ t + \Delta t\text{-}\varphi \rho h \theta r \Delta r\ t$$

This reduces to:

$$\frac{1}{r\Delta r}[\rho U_r \Delta r - r\Delta(\rho U_r)] = \frac{-\Delta(\varphi\rho)}{\Delta t},$$

And since

$$\Delta(\rho U_r)/\Delta r \to -\delta(\rho U_r)/\delta r$$

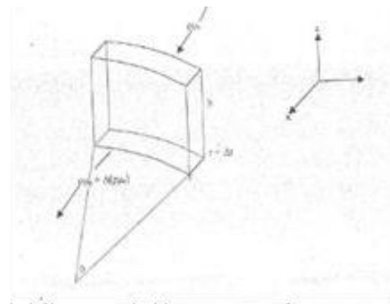

Fig. B- Radial element of porous medium used for mass

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho U_r) = -\frac{\delta(\varphi\rho)}{\delta t} \ldots\ldots\ldots\ldots (11)$$

where U_r is the volumetric flow rate per unit cross-section area in the radial direction.

Flow Laws

The Darcy's in field unit for a 3-D flow is given by:

$$U_x = -0.001127 \frac{K_x}{\mu}\frac{\delta p}{\delta x} \ldots\ldots\ldots\ldots (12a)$$

$$U_y = -0.001127 \frac{K_y}{\mu}\frac{\delta p}{\delta y} \ldots\ldots\ldots\ldots (12b)$$

$$U_z = -0.001127 \frac{K_z}{\mu}\left(\frac{\delta p}{\delta z} + 0.00694\rho\right) \ldots (12c)$$

After substituting these equations into the continuity equation, the result is

$$\frac{\partial(K_x \rho \delta p)}{\delta x\, \mu\, \delta x} + \frac{\partial(K_y \rho \delta p)}{\delta y\, \mu\, \delta y} + \frac{\delta}{\delta z}[\left(\frac{K_z}{\mu}\frac{\rho(\delta p)}{\delta z} + 0.000694\rho\right)] = \frac{1}{0.000264}\frac{\delta(\varphi\rho)}{\delta t} \ldots\ldots\ldots\ldots (13)$$

For slightly compressible liquids with constant compressibility, where c is defined by the equation

$$C = -\frac{1}{V}\frac{dV}{dp} = \frac{1}{\rho}\frac{d\rho}{dp} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

For constant compressibility, c, integration of Eqn.14 gives

$$\rho = \rho_o e^{C(P-P_o)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(15)$$

Putting Eqn. (15) into Eqn. (13), and with the assumptions stated above Eqn. (13) becomes

$$\frac{\partial[e^{C(P-P_o)}\frac{\partial p]}{\partial x}}{\partial x} + \frac{\partial[e^{C(P-P_o)}\frac{\partial p]}{\partial y}}{\partial y} + \frac{\partial[e^{C(P-P_o)}\frac{\partial p]}{\partial z}}{\partial z} = \frac{\emptyset\mu}{0.000264k}\frac{\partial[e^{C(P-P_o)}]}{\partial t}$$

Simplifying the above equation, yields

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - c\left[\frac{(\partial p)}{\partial x} + \frac{(\partial p)}{\partial y} + \frac{(\partial p)}{\partial z}\right] = \frac{\varphi\mu c}{0.000264k}\frac{\delta p}{\delta t} \ldots\ldots\ldots\ldots(16)$$

If it is assumed that c and $\frac{(\partial p)}{\partial x} + \frac{(\partial p)}{\partial y} + \frac{(\partial p)}{\partial z}$

then their product will become negligibly small, then Eqn. (16) becomes.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\varphi\mu c}{0.000264k}\frac{\delta p}{\delta t} \ldots\ldots\ldots\ldots(17)$$

For one-dimensional radial flow, the corresponding equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\frac{(r\partial p)}{\partial r} = \frac{\varphi\mu c}{0.000264k}\frac{\delta p}{\delta t} \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

The situation being analyzed is such that (1) pressure throughout the reservoir is uniform before production; (2) fluid is produced at a constant rate from. a single well of radius r_w centered in a· reservoir, and (3) there is no flow across the outer boundary (with radius r_e) of the reservoir. Stated mathematically, the initial and boundary conditions for the expression in Eqn. (17) are

At $t = 0, P = P_i$ for all r

At $r = r_e, q = 0$ for $t > 0$

Expressed in terms of dimensionless variables, the differential equations and its initial and boundary conditions become

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \ldots\ldots\ldots\ldots\ldots\ldots..(19)$$

$P_D = 0$ For all r_D at $t_D = 0$

$$\left.\frac{\partial P_D}{\partial r_D}\right| r_D = r_{De} \text{ For } t_D > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

$$\left.\frac{\partial P_D}{\partial r_D}\right| r_D = 1 \text{ For } t_D > 0 \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

D. Determination of Dimensionless well bore Pressures

So as to obtain dimensionless pressure solution for use in the analysis of pressure buildup tests, the effects of well bore phase redistribution will be incorporated into the diffusivity equation. For radial flow in an infinite, homogeneous isotropic reservoir of a fluid of small compressibility, this problem is stated m dimensionless variables as follows. From Eqn. (19), the diffusivity equation is

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \ldots\ldots\ldots\ldots\ldots\ldots..(19)$$

The boundary conditions are

$$P_D(r_D, 0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22b)$$

$$\lim_{r_D \to \infty} P_D(r_D, t_D) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22c)$$

$$-\frac{(\partial P_D)}{\partial r_D} = 1 - C_D\left(\frac{\partial P_{WD}}{\partial t_D} - \frac{\partial P_{XD}}{\partial t_D}\right) \ldots\ldots\ldots..(23)$$

$$P_D = [P_D - S(\partial P_D/\partial r_D)]_{r_D=1} \ldots\ldots\ldots\ldots\ldots(24)$$

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It has been shown that this problem also can be written as a convolution integral to account for well bore storage (Akintola *et al.,* 2015). This leads to Eqn. (25)

$$P_{WD}(t_D) = \int_0^{t_D}\{1 - C_D[\frac{\partial P_{WD}(\varphi)}{\delta t_D} - \frac{\delta P_{XD}(\varphi)}{\delta t_d}]\}.\frac{\partial P_D(t_{D-\varphi})}{\delta t_D} - \partial\varphi + S\left\{1 - C_D\left[\frac{\delta P_{WD}(t_D)}{\delta t_D} - \frac{\delta P_{XD}(t_D)}{\delta t_D}\right]\right\} \ldots\ldots (25)$$

When solved for

$$\Delta(P_{WD}) = \frac{[Z\Delta(P_D) + S][1 + C_D Z^2\,\Delta(P_{XD})]}{Z[1 + C_D Z\,(Z\Delta(P_D) + A)]} \ldots (26)$$

$\Delta(P_D)$has been expressed as in Eqn. (27) below by Van Everdingen and Hurst (1949)

$$\Delta(P_D) = \frac{K_O(\sqrt{z})}{z^{3/2}K_1(\sqrt{z})} \ldots\ldots (27)$$

At long times it has been shown that the above equation simplifies to a line source solution in Eqn. (28), as

$\sqrt{z}K_1(\sqrt{z}) \to 1$ When $s \to 0$ or$t_D \to \infty$.

$$\Delta(P_D) = 1/z\,.K_O\,(\sqrt{z}) \ldots\ldots (28)$$

The Laplace transform of $\Delta(P_{XD})$, the phase redistribution effect is:

$$\Delta(P_{XD}) = C_{xD}/z - C_{xD}/(z + 1/\lambda) \ldots\ldots (29)$$

Also, it has been observed that as$t_D \to \infty, z \to 0$, therefore,

$$K_O(\sqrt{z}) \to -\left[\ln\left(\sqrt{z/2}\right) + v\right] \ldots\ldots (30)$$

This gives Eqn. (31),

$$\Delta(P_D) = -1/z\left[\ln\left(\sqrt{z/2}\right) + v\right] \ldots\ldots (31)$$

$$\Delta(P_D) = \frac{\{[K_O(\sqrt{z}) + S][1 + C_D C_{XD} z^2(1/z - 1/(z + 1/\lambda_D))]\}}{(Z\{[1 + C_D Z[K_O(\sqrt{z}) + S]\})} \ldots (32)$$

The Laplace transform of Eqn. (12) for a line source well long-time approximation is given as Eqn. (33)

$$\Delta(P_{WD}) = \frac{([S - \ln\left(\sqrt{z/2}\right) - v][1 + C_D C_{XD} z^2(1/z + 1/v_D)])}{(z(1 + C_D z[S - \ln(\sqrt{z/2}) - v]))} \ldots (33)$$

In order to obtain the dimensional pressure for use m analyzing pressure buildup tests with phase redistribution, eqn. (33) must be inverted. Since this expression is too complicated for analytical inversion, the inverse Laplace transform will be calculated numerically by employing stehfest algorithm (Stehfest,1970). The computer program to do this calculation is given in the subroutine stefes.

E. The Derivative

The advantage of the derivative plot is that it is able to display in a single graph many separate characteristics that would otherwise require different plots (Rahman, 2012). The inclusion of the pressure derivative in the type curves and in the data makes the matching process much easier than if the function alone was used.

For an infinite acting radial cylindrical source well, the bottom hole shut-in pressure is estimated using eqn. (34),

$$P_{ws} = P_i + m\{\log 1688(\emptyset\mu C_t r_w^2/Kt) - 0.869S\}$$
$$= P_i + (m/2.303)\{-lnt + \ln(\varphi\mu C_t r_w^2/K) + 7.43 - 2S\} \ldots\ldots (34)$$

Upon differentiating the above equation with respect to time, we have,

$$\frac{dP_{ws}}{dt} = -\frac{m}{2.303}\left(\frac{1}{t}\right)$$
$$t\,x\,\frac{dP_{ws}}{dt} = -\frac{m}{20303} \ldots\ldots (35)$$

a constant as $t_D \to \infty$

But$t_D x \frac{dP_D}{dt_D} \to 0.5$, a constant as $t_D \to \infty$

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And the algorithm to handle the numerical differentiation eqn. (35) is shown below. Plots of P_{wD} & $t_D x\, dP_D/dt_D$ vs. dt_D are now made on a log- log graph and superimposed to generate the type- curves. Also, plots of ΔP_{ws} & $tx\, dP_{ws}/d_t$ vs.Δt for the pressure buildup data are made on a log-log paper having the same scale as the graph used for the type curves. This is then type-curve matched to obtain a match curve. A match point is now chosen to obtain reservoir permeability from Eqn. (36).

$$P_D = \frac{Kh\,(\Delta P_{ws})}{141.2 q\mu\beta}$$

$$K = \frac{141.2 q\mu\beta}{h}\left(\frac{P_D}{\Delta P_{ws}}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (36)$$

at match point

In calculating the pressure derivative some care is required, since the process of differentiating the data amplifies any noise that may be present. The best method to reduce the noise is to use data points that are separated by at least 0.2 of a log cycle, rather than points that are immediately adjacent. The numerical differentiation to achieve this is written below.

$$t\left({}^{\delta p}/_{\delta t}\right) = \left({}^{\delta p}/_{\delta lnt}\right)$$

$$= \left[\frac{\ln\left({}^{t_i}/_{t_{i-k}}\right)\delta P_{i+j}}{\ln\left({}^{t_{i+j}}/_{t_i}\right)\ln\left({}^{t_{i+j}}/_{i-k}\right)} + \frac{\ln\left({}^{t_{i+j}t_{i-k}}/_{t_i^2}\right)\delta P_I}{\ln\left({}^{t_{i+j}}/_{t_i}\right)\ln\left({}^{t_i}/_{i-k}\right)} + \frac{\ln\left({}^{t_{i+j}}/_{t_i}\right)\partial P_{i-k}}{\ln\left({}^{t_i}/_{t_{i-k}}\right) in\left({}^{t_{i+j}}/_{t_{i-k}}\right)}\right] \ldots (37a)$$

$$lnt_{i+j} - lnt_i \geq 0.2 \ldots\ldots (37b)$$

$$lnt_i - lnt_{i-k} \geq 0.2 \ldots (37c)$$

The value of 0.2 (known as the differentiation interval) could be replaced by smaller or larger values (usually between 0.1 and 0.5), with consequent differences in the smoothing of the noise.

F. COMPUTER APPLICATION

Subroutine stefes

This subroutine returns the value of the resulting integral, $P_{wD}\,(t_D)$. Subroutine stefes arrives at this solution by solving for the inverse Laplace transform using stehfest algorithm while maintaining double precision to improve accuracy.

Subroutine nurndif

This subprogram computes the numerical differentiation, and the resulting data are used to generate the derivative plot.

G. The Main Program

'The main program writes out the input data and the results obtained from the stehfest inversion of the function $\Delta(P_{wD})$. The main program also compares the values of Permeability, Slope, and Skin obtained by the conventional method and the use of the type curves and prints the output by writing them in a file.

IV.APPLICATION, RESULTS, INTERPRETATIONS & DISCUSSION

To illustrate the analysis of bottom hole pressure buildup surveys which are influenced by well bore phase redistribution as shown in the model developed, two actual field examples have been used from Walter and Fair (1981). The pressure buildup data with phase redistribution have been presented in Table 4.1 & Table 4.4 together with reservoir and fluid properties. The log-log plots of the pressure data vs. Homer time are also presented in Figs.7 and 8.

From the data plot in Fig. 2, a point on the unit slope straight line is estimated to be $\Delta P = 153\ psi$ at $\Delta t = 0.1\ hr$. The well bore storage coefficient is calculated as in Eqn. (36), and the apparent storage coefficient as in Eqn. (37):

$$C = {}^{A_w}/_{\rho_f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

$$= {}^{0.00387}/_{0.330}$$

$$= 0.01173 {}^{bbl}/_{psi}$$

$$C_{app} = {}^{q\beta\Delta t}/_{24\Delta p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

$$= 0.00635 {}^{bbl}/_{psi}$$

$= 0.00635 bbl/psi$

The type curves obtained are shown in Figs. 1- 8. Also Fig. 9 are type curves showing a comparison of dimensionless pressure with and without phase redistribution. When type curve matching, the actual buildup data were plotted on tracing paper, using the grid on the undistorted type curve as a plotting aid.

The gradient used in Eqn. (14) is calculated from flowing & static pressure surveys measured in conjunction with the buildup tests. Since $C_{app} > C$, one can conclude that the phase redistribution effect is significant. These values yield $C_D = 752$ and $C_{aD} = 407$. The data are then matched to the type curves for $C_{aD} = 400$ and $C_D = 750$ as shown in Fig. (7) with a match point chosen as $C_{xD} = 10, S = 0$, $t_D = 550$ at $0.4\ hour$, and $P_{wD} = 5.41$ at $\Delta P_w = 500$ psi. From the standard definitions of t_D and P_{wD}, the permeability is

calculated as follows:

For example, 1

$$K = \frac{141.2\ q\mu\beta P_D}{h(\Delta P_{ws})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

$$K = \frac{141.2\ x212x4x1.1x1.155}{10X102}$$

From P_{wD} match: K = 149.107 md

For example, 2

$$C = {}^{A_w}/_{\rho_f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (.40)$$

$$= {}^{0.00387}/_{0.330}$$

$$= 0.01173\ {}^{bbl}/_{psi}$$

$$C_{app} = {}^{q\beta\Delta t}/_{24\Delta p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

$$= 0.00635 {}^{bbl}/_{psi}$$

$$K = \frac{141.2\ q\mu\beta P_D}{h(\Delta P_{ws})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

$$K = \frac{141.2\ x14x4x1.05X1.83}{20X102}$$

From P_{wD} match: K = 7.45 md

The results agree closely with the estimates obtained by Walter and Fair (Walter and Fair,1981) and the results obtained by the semi log analysis techniques.

V. CONCLUSSIONS AND RECCOMENDATION

- CONCLUSIONS

Result shows that the mathematical model developed in this study by type curves are advantageous since they may allow test interpretation even when well bore storage distorts most or all of the test data; a feat which the conventional method cannot perform. Also, the inclusion of the pressure derivative in the type curves and in the data makes the matching procedure much easier. In this work, an exponential was used to represent the phase redistribution pressure function. It was observed that the observed storage constant often does not agree with that calculated from the well completion properties One possible explanation for this observation lies in the apparent storage observed to be associated with phase redistribution. The hump on the derivative plot is characteristic of a damaged well (i.e., one with positive skin). Another important observation on the derivative plot is that since the pressure in a buildup test stabilizes to a final value, hence the pressure derivative trends towards zero.

- RECOMMENDATION

It is recommended that the true and apparent storage coefficients always be calculated and checked for consistency before proceeding with the detailed analysis of a buildup survey.

The main assumption of exponential form used in this study to represent the phase redistribution pressure function has been known to represent phase redistribution in gas-lifted oil well very well. However, no meaningful experimental data are available to substantiate this completely. It is therefore recommended that such data be collected and they will prove useful either in verifying this function or in proposing a new function for the phase redistribution pressure. Although this work is based upon infinite, homogenous, radial reservoir model, the basic concepts have been found to be much general. It is therefore recommended that this technique be applied for the analysis of data in fractured system as well as other practical situations.

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