

# Adomian Decomposition Method Solution to Effect of Variable Viscosity on Mhd Mixed Slip Flow Near A Stagnation Point on A Nonlinearly Vertical Stretching Sheet in The Presence of Thermal Radiation and Viscous Dissipation

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**Abstract-** *In this study, effect of joule heating and variable fluid properties on magneto hydrodynamic (MHD) and thermally radiating slip-flow past a non-linearly stretching vertical sheet, subjected to uniform heat flux, is analyzed both semi-analytically and numerically. Similarity transformation is been used to transform the partial momentum and heat transfer equation into a set of non-linear ordinary differential equations. Using Adomian Decomposition Method alongside Runge-Kutta Fehlberg fourth-fifth order method with shooting technique.*

**Indexed Terms-** *Joule Heating, Similarity Transformation, Non-Linear Ordinary Differential Equations, Shooting Technique.*

## I. INTRODUCTION

In the early 20th century, (1904) Ludwig prardh a German engineer who is know as a pioneer that revolutionized the field of fluid mechanics gave an outstanding presentation (talk) entitled "On fluid flow with little friction", at the international Mathematics Congress in Heidelberg. In his presentation, he scrutinize the relationship between the viscosity of a fluid in contact with the very thin layer adjacent to the surface termed as the "boundary layer". This outstanding presentation given by Ludwig prardh brought about a revolution and more understanding that brought about development in fluid flow and the entire area of fluid mechanics.

The practical applications arising in industry and engineering problems know as problem of stagnation-point flow and heat transfer on stretching sheet arises in abundance, which are cooling of electronic devices, and nuclear reactors, polymer extrusion, drawing of plastic sheets, etc. Heat transfer of an in-compressible Newtonian viscous fluid over a stretching surface

placed in a porous medium which is a type of problem flow that is occurring in the polymer industry has received considerable attention in recent years because it is important. A flow induced by the flow problems with obvious relevance to polymer extrusion.

For illustration, in a process known as melt-spinning, generally the extrudate from the die is drawn, and stretched simultaneously into a Sheet or filament, Which by direct contact with water or Chilled metal rolls Solidifies rapidly through cooling or quenching (See Adeniyani, 2015). In Improving it's mechanical properties and the quality of the final product which greatly depends on the rate of cooling, stretching Impact unidirectional orientation to the extrudate ( Prased et al., 2010).

In recent times the study of MHD Stagnation point flow on stretching sheet has attracted many researchers from Various fields, and discussion as regards different aspects as been put in consideration (discussed), Including the pioneering work of Ishak et al. (2009) stretching sheet with variable surface temperature viscous dissipation analyzed by Shatey; and makinde (2013) and Khan et al. (2014), also Amon et al. (2013) 180 worked on the effect of slip along with Singh and Makinde (2014), and the analysis of the unsteady case, was studied by Shateyi and Marewo (2014).

Taking some instances, The effect of mixed convection due to a buoyancy force cannot be set aside for the vertical sheet, A different from the flow Induced by a stretching horizontal sheet. Also Some authors made contribution (analysis) on MHD with mixed convection boundary layer flow and heat transfer characteristic over a stretching Vertical

surface, the authors Ishak et al. (2010), Hayat and kazeem, (2011), Chamkha et al. (2013). In recent times, the study of MHD mixed convection Stagnation-point flow and heat transfer of an incompressible viscous fluid over a vertical stretching sheet was Considered by Ali et al. (2014). The flow over a linearly stretching vertical or horizontal Sheet were the investigations considered above, but the real Stretching velocity does not always need to be linear or uniform. Akyildiz et al. (2010) studied The Similarity solution of the boundary layer equations for a non-linearly stretching sheet. Also Akyildiz and Siginer (2010) investigated the flow and heat transfer over a non-linearly stretching sheet using a legendre Spectral method. Recently, Ashraf et al. (2015) Dhanai et al. (2015).

One of the early Pioneers to analyze the combined force and and free convection in boundary layer flow in the presence of non-linearly stretching sheet in Sparrow et al. (1959). There after some other authors (researchers) have investigated and extended various aspects of boundary-Layer flow of an electrically conducting fluid past a vertical or horizontal sheet with non-linear stretching velocity in the presence of transverse magnetic field.

The problem of flow and heat transfer in the boundary layer adjacent to a continuous moving surface has attracted many scholars because of its numerous applications in engineering or manufacturing processes namely continuous casting glass fibre production metal extrusion hot rolling of paper and textiles and wire drawing. The fiscal situation was recognised as a backward boundary layer problem by Sakiadis (1961). He was the first among others to investigate the flow behaviour for this class of boundary layer problems. In his pioneering works solutions were obtained to the boundary layer flows on continuous moving surfaces which are substantially different from those of boundary layer flows on stationary surfaces. The thermal behaviour of the problem was studied by Erikson et Al. (1966) using finite difference and integral methods and experimentally verified by Tsou et Al. (1967). Thereafter various aspects of the above boundary layer problem on continuous moving surface were considered by many researchers (Crane (1970), Grubka and Bobba (1985), Vleggar ,(1977),

Soundalgekar and Ramana Murthy (1980), Gupta and Gupta (1977), Chen and Char(1988), Chen and Strobel (1980)]. Sparrow et Al. (1959) analysed the combined forced and free convection in boundary layer flow and considered non-linearly stretching sheet. Since then there has been increasing interest in investigating the mhd with mixed convection boundary layer flow and heat transfer over is stretching vertical surface. Ishak et Al. (2009). Carried out analysis on mhd stagnation point flow towards a stretching sheet. Suali et Al. (2012) investigated the unsteady two dimensional stagnation point flow and heat transfer hoover is stretching or shrinking sheet with prescribed surface heat Flux. Khan et Al. (2014) analysed the image d stagnation point ferrofluid flow and heat transfer to what is stretching sheet. Shateyi and Makinde (2013) investigated the hydromagnetic stagnation point flow towards radially stretching convectively heated disk ie the study of stagnation point flow and heat transfer of an electrically conducting incompressible viscous fluid with convective boundary conditions. The mhd boundary layer flow over a vertical stretching or shrinking sheet in a nanofluid was investigated buy Makinde et Al. (2013). Pal and Mondal (2014) presented in numerical model to study the effects of temperature dependent viscosity and variable thermal conductivity on mixed convection problem. Sharma et Al. (2014) numerically studied the mhd stagnation point flow of a viscous incompressible and electrically conducting fluid over a stretching or shrinking permeable semi-infinite flat plate. Hsiao (2011) studied energy problems of conjugate conduction convection and radiation heat and mass transfer with viscous dissipation and magnetic effects. as discussed dissipation is the process of converting mechanical energy of downward flowing water into thermal and acoustical energy health devices are designed in streambeds for example to reduce the kinetic energy of flowing Waters. The basics important is meant to reduce their erosive potential on banks and river bottoms. Vajravelu and Dadjinicalaou (1993) studied heat transfer characteristics over-stretching surface with viscous dissipation. Shen et Al. (2015) study the problem of mhd mixed convention flow over a stagnation point region over a nonlinear stretching sheet with sleep velocity and prescribed surface heat Flux. Just and Chaudhary (2008) investigated the problem of mhd boundary layer flow over a stretching sheet for stagnation point heat transfer with and

without viscous dissipation and Joule heating. Abel et Al. (2011) investigated numerically the problem of mhd flow and heat of an incompressible viscous fluid with the presence of buoyancy Force and viscous dissipation. Ali (2013) studied heat transfer and flow field characteristics on a stretched permeable surface subject to mixed convection. Thermal radiation effect is very important in process involving high temperatures and in space technology. Recently much attention has been focused on the convection as a mode of energy transfer in a hypersonic flights, missile reentry gas-cooled nuclear reactors and rocket combustion Chambers. Shateyi (2008) investigated thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Machireddy (2014) study the effects of Joule heating on steady mhd mixed convection boundary layer flow over a stretched vertical flat plate in the presence of thermal radiation and viscous dissipation. Chaudhary and Kumar (2013) analysed the study two-dimensional in which the boundary layer flow of a viscous incompressible and electrically conducting fluid near stagnation point past a shrinking sheet with suction conditions. Mukhopadhyay (2013) analysed the effect of partial slip on boundary layer over a non-linearly stretching surface with suction or injection. In most of the above-mentioned literature is the thermal physical properties of the ambient fluid were assumed to be constant. However it is also well known that this physical properties may change with temperature especially for fluid viscosity and thermal conductivity. For lubricating fluids heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid and the properties of the field and no longer assumed to be constant. The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer and so the heat transfer at the wall is also affected. There for to predict the flow and heat transfer rates it is necessary to take into account the variable field properties. Motivated by the above studies the current seminal work aims to numerically analyse the effect of Joule heating and variable fluid properties on mhd and radiating slip flow past a nonlinear stretching vertical sheet full stop the partial momentum and if transfer equation are transformed into a set of ordinary differential equations by employing suitable similarity

transformations. Using the wrong you cook a perfect fit other method finite difference method differential transfer method multiplication method adomian decomposition method numerical calculations to the desired level of accuracy obtained for different values of dimensionless parameters full-stop the results are presented graphically and in tabular form the results for special cases are also compared to those by S. In this work, we consider steady, laminar incompressible flow and the flow is of two dimensions. The effect of electric field Hall current is negligible due to the fact that the non-linearly stretching plate is assumed to be electrically neutral. The magnetic field is much stronger than the induced magnetic field because the magnetic Reynolds number is very small. The radiation heat flux is assumed to obey Rosseland approximation.

Effects of temperature variation and other parameter are investigated. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid, and the properties of the fluid are no longer assumed to be constant.

The increase in the transport phenomena is caused by the corresponding increase in temperature by reducing the physical properties across the thermal boundary layer, and so the heat transfer at the wall is also affected. Therefore, to predict the flow and heat transfer rates, it is necessary to take into account the variable fluid properties.

The influences of suction and injection on the wall were investigated. The boundary conditions used by Shateyi and Mabrod (2015) were employed.

## II. MATHEMATICAL ANALYSIS

### Mathematical formulation

This section, we consider the steady two-dimensional effect of joule heating and variable fluid properties on MHD and radiating slip-flow past a non-linear stretching vertical sheet. We also consider a prescribed surface heat flux. We confine the flow to a region  $y \geq 0$ , where  $y$  is the coordinate measured perpendicular to the stretching surface. A uniform magnetic field of stretch  $B(x)$  is applied in the direction normal to the surface (see fig. 1). The sheet stretching velocity is

assumed to be  $u_w(x) = cx^m$ , and the external velocity is prescribed as  $u_e(x) = ax^m$ , where  $c$  and  $a$  are positive constants. While  $m$  is the nonlinearity parameter, with  $m = 1$  for the linear case and assumed to be  $u_w(x) = cx^m$ , and the external velocity is prescribed as  $u_e(x) = ax^m$ , where  $c$  and  $a$  are

positive constants. While  $m$  is the nonlinearity parameter, with  $m=1$  for the linear case and  $m \neq 1$  for the nonlinear case. Under the above assumptions and the boundary layer and Boussinesq approximation, the governing equations for the current study are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left[ \mu(T) \frac{\partial u}{\partial y} \right] + \frac{\sigma B^2(x)}{\rho} (u_e - u) + g\beta(T - T_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial T}{\partial y} \left[ k(T) \frac{\partial T}{\partial y} \right] + \frac{\mu(T)}{\rho c_p} \left[ \frac{\partial u}{\partial y} \right]^2 - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{\sigma B^2(x)}{\rho c_p} (u_e - u)^2 \quad (3)$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions, respectively,  $\nu$  is the kinematic viscosity,  $\rho$  is the fluid density,  $\sigma$  is the electrical conductivity,  $B(x)$  is the transverse magnetic field,  $g$  is the acceleration due to gravity,  $\beta$  is the thermal expansion coefficient,  $T$  is the fluid temperature,

$\alpha$  is the thermal diffusivity,  $c_p$  is the heat capacity at constant pressure,  $\mu$  is the dynamic viscosity,  $q_r$  is the radiative heat flux. The associated boundary conditions to the current model are given by:

$$y = 0: u = u_w(x) + \frac{2-\delta_v}{\delta_v} \lambda_0 \frac{\partial u}{\partial y}, v = v_w(x), \frac{\partial T}{\partial y} = -\frac{q_w(x)}{k} \quad (4a)$$

$$y \rightarrow \infty: u \rightarrow u_e(x), T \rightarrow T_\infty \quad (4b)$$

where  $\sigma_{\nu}$  is the tangential momentum accommodation coefficient,  $\lambda_0$  is the mean free path,  $v_w(x)$  is the suction injection velocity,  $k$  is the thermal conductivity, and  $q_w$  is the surface heat

flux. By using the Rosseland diffusion approximation (Hossain et al. (2001), Raptis (1998)) among other researchers, the radiative heat flux,  $q_r$  is given by

$$q_r = -\frac{4\sigma^* T_\infty^3}{3K_s} \frac{\partial T^4}{\partial y}, \quad (4c)$$

The use of Taylor's theorem for the expansion of temperature dependent function  $F(T)$  about the free stream temperature  $T_\infty$  tends itself to the smallness

of temperature difference  $T - T_\infty$  consequent upon which

$$F(T) = F(T_\infty) + (T - T_\infty)F'(T_\infty) + \frac{1}{2}(T - T_\infty)^2 F''(T_\infty) + \dots$$

On replacing  $F(T)$  by  $T^4$  and neglecting higher order terms, one obtains

$$T^4 \approx T_\infty^3 (4T - 3T_\infty) \text{ Using } q_r = \frac{-4\sigma^* T_\infty^3}{3K_s} \frac{\partial T^4}{\partial y}, \text{ we find } q_r = \frac{-16\sigma^* T_\infty^3}{3K_s} \frac{\partial T}{\partial y}, \frac{\partial q_r}{\partial y} = \frac{-16\sigma^* T_\infty^3}{3K_s} \frac{\partial^2 T}{\partial y^2} \text{ (see Adeniyani. (2015))}$$

### III. METHODOLOGY

Following Shateyi and Mabood (2015) we introduce the following similarity transformations:

$$\eta = \sqrt{\frac{a}{v}} y x^{\frac{m-1}{2}}, \psi = \sqrt{a v x} \frac{m+1}{2} f(\eta), \theta = \sqrt{\frac{a}{v}} \frac{k(T-T_\infty)}{q_0 x^{2m-1}} \quad (5)$$

Here  $\psi$  is the stream function such that  $u = \frac{\partial\psi}{\partial y}$ ,  $v = -\frac{\partial\psi}{\partial x}$  and hence continuity equation will be satisfied.

From (2) and (3), using the similarity transformation.

Here the momentum equation is given to be

$$f'''(\eta) - \frac{a'}{a'\theta+1}\theta'(\eta)f''(\eta) + (1+a'\theta)M(1-f'(\eta)) + (1+a'\theta)m(1-f'^2(\eta)) + (1+a'\theta)f(\eta)f''(\eta)\left(\frac{m+1}{2}\right) + (1+a'\theta)\lambda \cdot \theta = 0 \quad (6)$$

And the energy equation is given as

$$\frac{1}{Pr}\left(\gamma\theta'^2(\eta)N1 + \gamma\theta(\eta)^{N-1} + \theta''(\eta)1 + \gamma\theta(\eta)^N + \frac{4}{3}R\theta''(\eta)\right) + Ec f'^2(\eta) + M \cdot Ec(1-f'(\eta))^2 - f'(\eta)\theta(\eta)(2m-1) + \theta'(\eta)\left(\frac{m+1}{2}f(\eta)\right) = 0 \quad (7)$$

The corresponding boundary conditions to the transformed equation are:

$$f(0) = f_w, \quad f'(0) = \epsilon + \delta f''(0), \quad \theta'(0) = -1 \quad (8a)$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0 \quad (8b)$$

Where  $\epsilon$  is the velocity ratio and  $\delta$  is the velocity slip.

#### IV. METHOD OF SOLUTION

In this paper, an efficient Semi-analytical method (Adomian decomposition method) and numerical scheme Runge-Kutta Fehlberg fourth fifth order method as been employed to investigate the flow model defined by the equations (2) - (3) with the boundary equations for different values of controlling parameters. The formula for the the Runge-Kutta-Fehlberg fourth fifth order numerical method is given below ( see Shateyi and Mabood, 2015):

Adomian Decomposition Method (ADM)

The decomposition method was introduced by Adomian [27]. Consider the general equation:  
 $\varphi[u(y)] = g(y) \quad (9)$

Where  $\varphi$  represents a general non-linear ordinary (or partial) differential operator involving both linear and non-linear terms. The linear terms is decomposed into the form  $L + R$ , where  $L$  is usually taken as the highest order derivative which is assumed to be easily invertible and  $R$  is the linear differential operator of order less than  $L$ . Therefore, equation (9) can be expressed as

$$Lu + Ru + Nu = g(y) \quad (10)$$

Where  $Nu$  represents the non-linear terms of  $\varphi[u]$ . Applying the inverse operator,  $L^{-1}$  to both sides of equation (10) gives

$$u = L^{-1}g - L^{-1}(Ru + Nu) \quad (11)$$

If  $L$  is a third order operator, then  $L^{-1}$  is a 3-fold integral. Now, solving equation (11), we have

$$u = \sum_{j=0}^2 \alpha_j \frac{y^j}{j!} + L^{-1}g - L^{-1}(Ru + Nu),$$

Where  $\alpha_j(j = 1..3)$  are constants of integration and can be determined from the given boundary conditions.

The standard Adomian decomposition method defines the solution  $u$  by the infinite series

$$u = \sum_{n=0}^{\infty} u_n$$

and the non-linear term by the infinite series

$$Nu = \sum_{n=0}^{\infty} A_n$$

where  $A_n$  are the Adomian polynomials determined formally from the relation;

$$A_n = \frac{1}{n!} \left[ \frac{d^n}{d\lambda^n} N \left( \sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, \quad i = 0,1,2,3, \dots$$

The  $u_n$  are determined from the recursive algorithm

$$v = \sum_{j=0}^2 \alpha_j \frac{y}{j!} + L^{-1} g v_{n+1} = -L^{-1} (Rv + Nv) n \geq 0,$$

where  $u_0$  is the zeroth component. For numerical computation, the truncated series solution is obtained as

$$S_n(y) = \sum_{k=0}^{n-1} u_k$$

where  $S_n$  denotes the  $n$  – term approximation of  $u(y)$ .

### Solution of Problem

In [29] and [31] The Method was used by Mustapha and Salau. Applying ADM to the equation (6) and (7) subject to the boundary condition (8a) & (8b) is obtained via ADM.

The method is imperative because of the non-linearity involved. Equations (6 – 7) can be written in operator form Where  $L_1 = \frac{d^3}{a\eta^3}$  and  $L_2 = \frac{d^2}{a\eta^2}$  is a

fourth order and second order differential operator respectively, with inverse operators  $L_1^{-1} = \int_0^\eta \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau d\tau$  and  $L_2^{-1} = \int_0^\eta \int_0^\eta (\cdot) d\tau d\tau$  respectively.

In terms of Adomian decomposition methods  $f(\eta)$  and  $\theta(\eta)$  are assumed to be a series solution of the form

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) \text{ and } \theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta)$$

And the non-linear terms are decomposed as series Where Adomian polynomials are generated

The recursive relations for the approximate analytical solution of system (8-10) are given as:

$$\begin{aligned} f_0 &= f_w + \varepsilon \eta \left( \frac{\eta^2}{2} + \delta \eta \right) \alpha_1 + \frac{\eta^3}{6} \alpha_2, \\ \theta_0 &= \alpha_3 - \eta \end{aligned} \quad (35)$$

The following partial sum

$$f(\eta) = \sum_{n=0}^{\infty} f_k(\eta) \text{ and } \theta(\eta) = \sum_{n=0}^{\infty} \theta_k(\eta)$$

Are the approximate solutions. The solution to the fluid flow are coded using algebraic symbolic package called Maple 2020 .

### Runge-Kutta-Fehlberg fourth fifth order numerical method

$$\begin{aligned} k_0 &= f(x_i, y_i), \\ k_1 &= f\left(x_i + \frac{1}{4}h, y_i + \frac{1}{4}hk_0\right), \\ k_2 &= f\left(x_i + \frac{3}{8}h, y_i + \left(\frac{3}{32}k_0 + \frac{9}{32}k_1\right)h\right), \\ k_3 &= f\left(x_i + \frac{12}{13}h, y_i + \left(\frac{1932}{2197}k_0 - \frac{7200}{2197}k_1 + \frac{7296}{2197}k_2\right)h\right), \\ k_4 &= f\left(x_i + h, y_i + \left(\frac{439}{216}k_0 - 8k_1 + \frac{3860}{513}k_2 - \frac{845}{4104}k_3\right)h\right), \\ k_5 &= f\left(x_i + \frac{1}{2}h, y_i + \left(-\frac{8}{27}k_0 + 2k_1 - \frac{3544}{2565}k_2 + \frac{1859}{4104}k_3 - \frac{11}{40}k_4\right)h\right), \\ y_{i+1} &= y_i + \left(\frac{25}{216}k_0 + \frac{1408}{2565}k_2 + \frac{2197}{4104}k_3 - \frac{1}{5}k_4\right)h, \\ z_{i+1} &= z_i + \left(\frac{16}{135}k_0 + \frac{6656}{12825}k_2 + \frac{28561}{56430}k_3 - \frac{9}{50}k_4 + \frac{2}{55}k_5\right)h, \end{aligned}$$

Where  $y$  is the fourth- order Runge-Kutta and  $z$  is the fifth-order Runge-Kutta. An estimate of the error can be obtained(calculated) by subtracting the two values obtained. If the error exceeds a specific threshold, the the results obtained can be recalculated using a smaller "step size". The approach to estimating the new step size is given below as :

$$h_{new} = h_{old} \left( \frac{\varepsilon h_{old}}{2 | z_{i+1} - y_{i+1} } \right)^{\frac{1}{4}}$$

Numerical computations were carried out with  $\nabla \eta = 0.01$  The software package in Maple. The sloe requirement that the variation of the dimensionless velocity and temperature is less than  $10^{-6}$  between any two successive iterations is employed as the criterion for convergence. The asymptotic boundary

conditions were approximated by using a value of 25 for  $\eta_{max}$  as follows:

$$\eta_{max} = 25, f'(25) = 1, \theta(25) = 0.$$

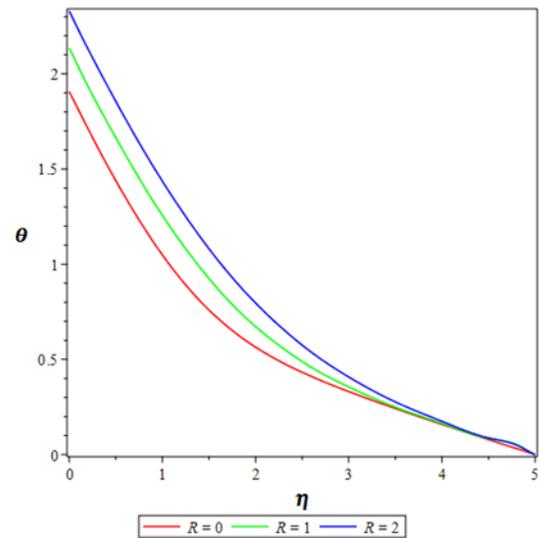
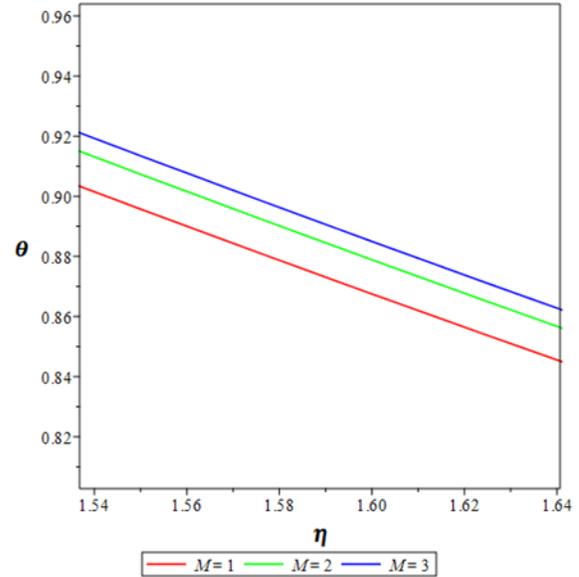
Which make it possible that all numerical solutions approached the asymptotic values correctly. The effects of emerging parameters on the dimensionless velocity, temperature, skin friction coefficient and the rate of heat transfer are investigated.

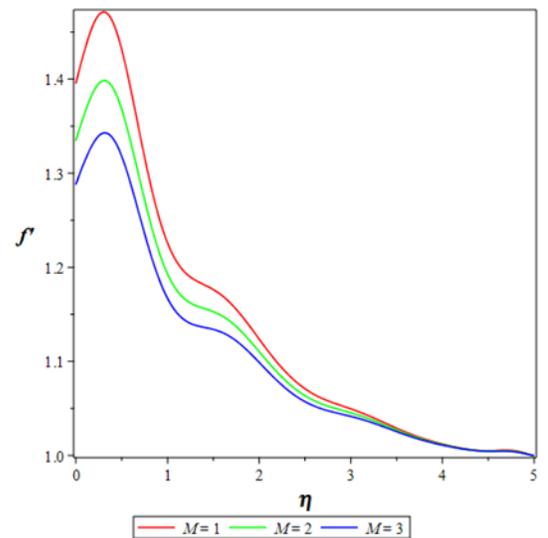
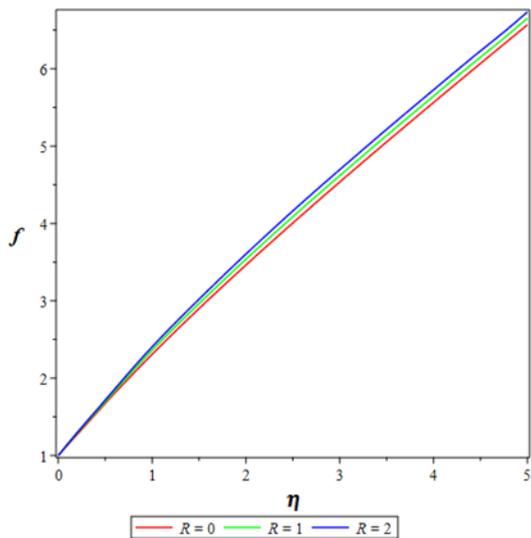
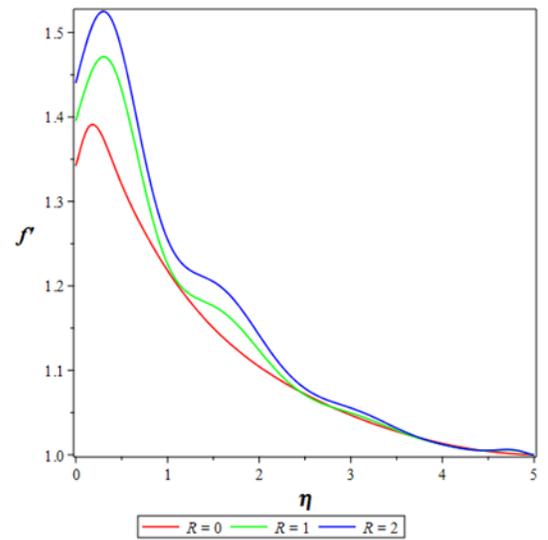
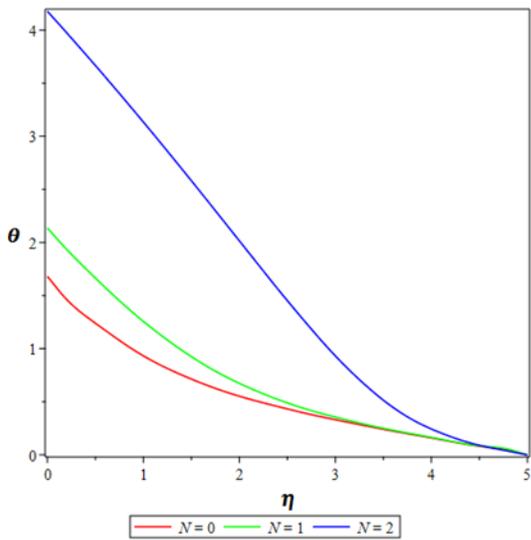
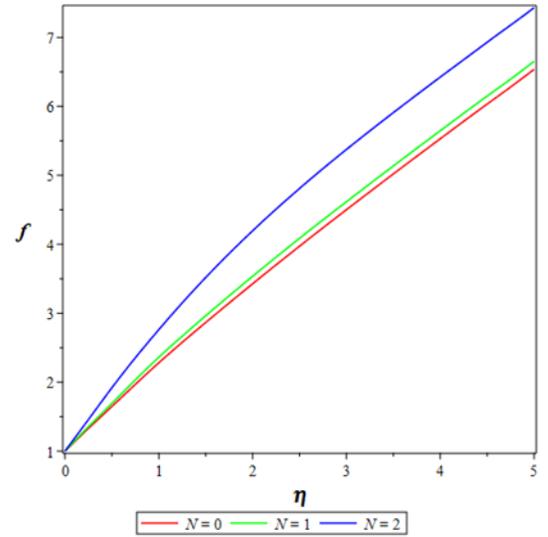
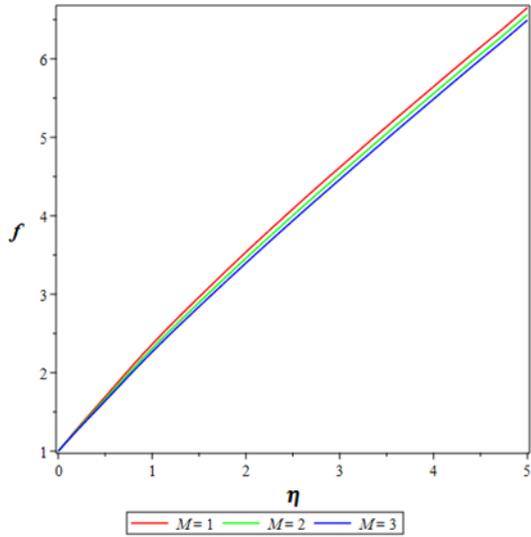
To validate the present solution, comparison has been made with previously published data from the literature for skin friction coefficient and heat transfer rate in Table 1. In Table 2 we have provided comparison of  $f''(0)$  values with other published results and they are found to be in excellent agreement. Table 3 presents the effects of various parameters on skin friction coefficient and heat transfer rate. It is clearly seen that with  $R$  and  $Ec$  heat transfer rate is decreasing whereas for  $f_w$  and  $m$  it is increasing.

#### V. NUMERICAL RESULT AND DISCUSSION

In this chapter, a semi-analytic and numerical parametric study is taken into consideration and the results are presented in tabular and graphical forms. For this particular problem, stretching greatly reduces the skin friction on the wall surface but therefore increases the rate of heat transfer as can be clearly seen in Table 1. Fluid injection leads to the reduction of the skin friction as can be observed in Table 3. However, the rate of heat transfer increases as the values of injection parameter increase. By increasing the nonlinearity parameter ( $m$ ) has a very insignificant effect on the skin friction as well as on the Nusselt number. In Table 3, we also observe that the thermal radiation parameter increases the values of the skin friction but greatly reduces the values of the Nusselt number.

The graphical representations of the numerical results are illustrated in the figures that precedes.





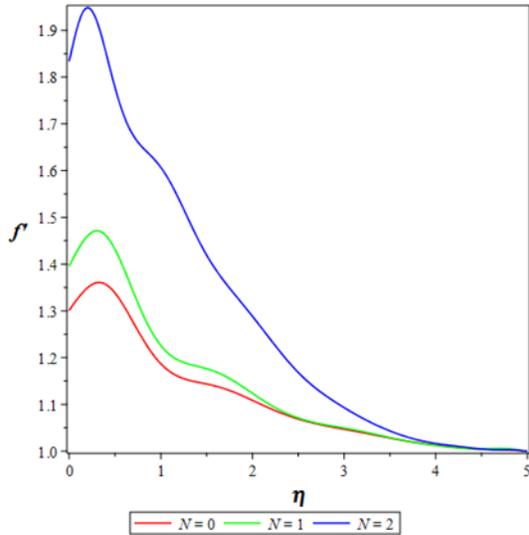


Table 1. Comparison of Skin friction coefficient and Nusselt number for different values of when

$\varepsilon$	Shen et al (2015)		Shateyi and Mabood (2016)		Numerical Result		ADM (Present Results)	
	$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$
0.1	0.6599	2.1772	0.659878	2.176819	0.63570	0.63570	2.17683	2.17683
1	-0.0248	2.2670	-0.024792	2.266589	-0.02647	2.26491	-0.02647	2.26491
2	-0.7945	2.3593	-0.794489	2.358832	-0.77363	2.37266	-0.77363	2.37266

Table 2. Comparison of the values of  $f''$  with those of Wang (2008), Wong et al. (2013) and Shateyi and Mabood when

$\varepsilon$	Wang(2008)	Wong et al.	Shateyi and Mabood	Numerical result	ADM(Present Results)
0	1.232588	1.23259	1.232587	1.2325875	1.2325875
0.1	1.14656	1.14656	1.146561	1.1465610	1.1465610
0.2	1.05113	1.05113	1.051129	1.0511300	1.0511300
0.5	0.71330	0.71329	0.713295	0.7132948	0.7132948
1	0	0	0	$-3.219902 \times 10^{-31}$	$-3.219902 \times 10^{-31}$
2	-1.88731	-1.88730	-1.887307	-1.8873069	-1.8873069
5	-10.26475	-10.26475	-10.264749	-10.2647493	-10.2647493

Table 3. Comparison of the values of skin friction coefficient and Nusselt Number with Shateyi and Mabood for different values when

					Shateyi and Mabood (2016)		Numerical result		ADM(Present Results)	
$f_w$	$m$	$\lambda$	R	Ec	$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$

1	1	1	1	1	0.097006	1.11964	0.097349	1.116683	0.097349	1.116683
1	3	1	1	1	0.021821	2.14261	0.023992	2.144588	0.023992	2.144588
1	1	-0.5	1	1	-0.05357	1.06907	-0.053624	1.068107	-0.053624	1.068107
1	1	1	3	1	0.180387	0.72148	0.181639	0.717405	0.181639	0.717405
1	1	1	1	10	0.099332	1.10272	0.103439	1.066949	0.103439	1.066949

Table 4. Computations showing Values of skin friction coefficient and Nusselt Number for Various values of basic flow parameters

$\gamma$	Ec	N	$a'$	Pr	$\delta$	$\varepsilon$	$f_w$	m	$\lambda$	R	M	Numerical result		ADM(Present Results)	
												$f''(0)$	$\frac{1}{\theta(0)}$	$f''(0)$	$\frac{1}{\theta(0)}$
1	1	1	1	1	1	1	1	1	1	1	1	0.207537	0.83827	0.207537	0.83827
2	1	1	1	1	1	1	1	1	1	1	1	0.329306	0.62634	0.329306	0.62634
3	1	1	1	1	1	1	1	1	1	1	1	0.488111	0.48684	0.488111	0.48684
0.5	1	1	1	1	1	1	1	1	1	1	1	0.165406	0.97017	0.165406	0.97017
0.5	2	1	1	1	1	1	1	1	1	1	1	0.168345	0.95962	0.168345	0.95962
0.5	3	1	1	1	1	1	1	1	1	1	1	0.171511	0.94855	0.171511	0.94855
0.5	1	0	1	1	1	1	1	1	1	1	1	0.132352	1.12089	0.132352	1.12089
0.5	1	0.5	1	1	1	1	1	1	1	1	1	0.146283	1.05101	0.146283	1.05101
0.5	1	2	1	1	1	1	1	1	1	1	1	0.249546	0.74463	0.249546	0.74463
0.5	1	1	1.5	1	1	1	1	1	1	1	1	0.183372	0.97011	0.183372	0.97011
0.5	1	1	2	1	1	1	1	1	1	1	1	0.198076	0.97068	0.198076	0.97068
0.5	1	1	3	1	1	1	1	1	1	1	1	0.221641	0.97088	0.221641	0.97088
0.5	1	1	1	0.72	1	1	1	1	1	1	1	0.283226	0.70680	0.283226	0.70680
0.5	1	1	1	2	1	1	1	1	1	1	1	0.048721	1.94244	0.048721	1.94244
0.5	1	1	1	5	1	1	1	1	1	1	1	0.008922	4.96053	0.008922	4.96053
0.5	1	1	1	1	0	1	1	1	1	1	1	0.844051	0.93544	0.844051	0.93544
0.5	1	1	1	1	1	1	1	1	1	1	1	0.165790	0.96930	0.165790	0.96930
0.5	1	1	1	1	2	1	1	1	1	1	1	0.092263	1.03045	0.092263	1.03045
0.5	1	1	1	1	1	0.1	1	1	1	1	1	0.889455	0.93183	0.889455	0.93183
0.5	1	1	1	1	1	0.5	1	1	1	1	1	0.566179	0.95412	0.566179	0.95412
0.5	1	1	1	1	1	0.8	1	1	1	1	1	0.325525	0.96503	0.325525	0.96503
0.5	1	1	1	1	1	1	-2	1	1	1	1	0.493255	0.47517	0.493255	0.47517
0.5	1	1	1	1	1	1	-1	1	1	1	1	0.401955	0.59407	0.401955	0.59407
0.5	1	1	1	1	1	1	-0.5	1	1	1	1	0.341352	0.66539	0.341352	0.66539
0.5	1	1	1	1	1	1	0	1	1	1	1	0.275471	0.75252	0.275471	0.75252
0.5	1	1	1	1	1	1	0.5	1	1	1	1	0.215790	0.85345	0.215790	0.85345
0.5	1	1	1	1	1	1	1	1	1	1	1	0.165932	0.96899	0.165932	0.96899
0.5	1	1	1	1	1	1	1	2	1	1	1	0.061305	1.53579	0.061305	1.53579
0.5	1	1	1	1	1	1	1	3	1	1	1	0.033406	1.98710	0.033406	1.98710
0.5	1	1	1	1	1	1	1	4	1	1	1	0.021403	2.38657	0.021403	2.38657
0.5	1	1	1	1	1	1	1	1	-1	1	1	-1.036637	0.36422	-1.036637	0.36422
0.5	1	1	1	1	1	1	1	1	0.5	1	1	0.086291	0.95423	0.086291	0.95423

0.5	1	1	1	1	1	1	1	1	1.5	1	1	0.240570	1.02025	0.240570	1.02025
0.5	1	1	1	1	1	1	1	1	1	2	1	0.248194	0.76732	0.248194	0.76732
0.5	1	1	1	1	1	1	1	1	1	3	1	0.321580	0.65738	0.321580	0.65738
0.5	1	1	1	1	1	1	1	1	1	4	1	0.449447	1.87020	0.449447	1.87020
0.5	1	1	1	1	1	1	1	1	1	1	2	0.140858	0.96017	0.140858	0.96017
0.5	1	1	1	1	1	1	1	1	1	1	3	0.122188	0.95429	0.122188	0.95429
0.5	1	1	1	1	1	1	1	1	1	1	5	0.094810	0.94710	0.094810	0.94710

### SUMMARY

In this study, effect of joule heating and variable fluid properties on magneto hydrodynamic(MHD) and thermally radiating slip-flow past a non-linearly stretching vertical sheet, subjected to uniform heat flux is analyzed, both semi-analytical and numerical methods that can be applied to solve the non-linear differential equations were considered and explained in details, in this case we consider Adomian decomposition method and Runge-Kutta-Fehlberg fourth fifth order numerical method to obtain solutions for momentum and energy equations.

### CONCLUSION

In this paper, the following conclusions were made. The velocity field is enhanced by increasing values of temperature ratio. This in turn enhances the skin friction coefficient with little effect.

The effects of temperature ratio on temperature profile is observed, increasing the temperature ratio lead or causes the fluid temperature to decrease.

The velocity field is enhanced by increasing values of temperature exponent. Thus in turn suppresses the skin friction coefficient.

The effects of temperature exponent on temperature profile is observed, increasing the temperature exponent lead or causes the fluid temperature to increase.

Velocity field is enhanced by increasing values of temperature variation. Thus in turn suppresses the skin friction coefficient.

The effects of temperature variation on temperature profile is observed, increasing the temperature

variation lead or causes the fluid temperature to increase.

In some cases, different flow behaviors were observed with opposing and assisting flows under different parameters.

### Contribution to knowledge

In this paper the extension of [16] was considered by looking the the temperature ratio effect, temperature exponent and variation which shows a tremendous effect on the fluid flow.

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