

Optimization of Sandcrete Brick Mix Proportions Using Scheffe's Simplex Lattice Design for Enhanced Compressive Strength and Moisture Resistance

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Abstract- Sandcrete bricks, while widely used in affordable housing across developing countries, suffer from inherent limitations including low compressive strength and high moisture absorption. This study addresses these challenges by optimizing sandcrete brick mix proportions incorporating laterite and water treatment sludge (WTS) using Scheffe's (5,3) simplex lattice design. The research characterized laterite (46.2% passing 0.075 mm sieve, maximum dry density of 2.03 Mg/m³, optimum moisture content of 13.5%, and California Bearing Ratio of 90%), cement (28% consistency, 166/339 minutes initial/final setting time, 3.0 mm expansion, and 84.4 MPa compressive strength at 28 days), and fine aggregates (specific gravity 2.82, water absorption 2%). Thirty-five experimental mixes were formulated and tested for compressive strength and water absorption at 7, 14, and 28 days. Compressive strength results ranged from 0.29 MPa to 24.49 MPa at 28 days, with over 280 mixes exceeding the 7 MPa threshold suitable for structural applications. Water absorption ranged from 0.03% to 3.37%, with an inverse relationship observed between porosity and strength. The optimization model yielded a high coefficient of determination ($R^2 = 99.24\%$), confirming excellent predictive capability. The optimized mix proportions (water 0.560, cement 1.070, WTS 0.651, laterite 0.549, and fine aggregate 2.761) achieved a maximum compressive strength of 14.99 MPa. The study concludes that laterite and WTS, when optimally proportioned using Scheffe's simplex lattice design, significantly enhance sandcrete brick performance while promoting sustainable construction practices through waste utilization.

Keywords: Sandcrete Bricks, Scheffe's Simplex Lattice Design, Laterite, Water Treatment Sludge

I. INTRODUCTION

Sandcrete bricks are widely utilized as a primary construction material in many developing countries, particularly for affordable housing projects, due to their cost-effectiveness and ease of production. However, despite their widespread use, sandcrete bricks face inherent limitations, including low

compressive strength and high moisture absorption, which can compromise their structural integrity and durability, particularly in regions with high humidity or adverse environmental conditions (Ikwaakor *et al.*, 2019). This issue, coupled with the increasing demand for sustainable building materials, presents a critical challenge to the construction industry. Improving the compressive strength and moisture resistance of sandcrete bricks is vital to enhance their performance and ensure they meet the structural demands of modern construction, particularly in low-cost housing (Oyebanji *et al.*, 2020). The real-world challenge lies in improving the mechanical and durability properties of sandcrete bricks while maintaining or reducing their production costs. The vulnerability of traditional earthen construction to environmental factors, particularly during rainy seasons, has been highlighted by Ndububa and Mukaddas (2016), who documented widespread mud house failures in Bauchi, Nigeria, with approximately 76% of surveyed structures experiencing wall collapses. Traditional sandcrete mixtures consist of water, cement, fine aggregate, and sand, but the optimization of these materials has often been conducted through trial-and-error methods or simple empirical formulas, which are both inefficient and labor-intensive. As a result, there is a growing need for systematic and scientifically grounded approaches to determine the optimal proportions of materials that would enhance the properties of sandcrete bricks (Okafor & Ugwu, 2018). Thus, there is a compelling need to explore alternative and locally available materials that can enhance the performance of sandcrete bricks while promoting environmental sustainability.

In this context, the incorporation of laterite and water treatment sludge (WTS) into sandcrete bricks has garnered attention as a potential solution.

Laterite, a type of soil abundant in tropical regions, is known for its pozzolanic properties, which make it suitable for use in cementitious applications. Water treatment sludge, a by-product of the water purification process, is typically disposed of as waste. However, it has been suggested that when properly processed, WTS can serve as a valuable material for improving the physical properties of concrete and sandcrete bricks (Ikwaakor *et al.*, 2019; Adedeji & Olutegun, 2021). Despite these advancements, the systematic optimization of sandcrete brick mixes that incorporate both materials remains underexplored. The use of Scheffe's simplex lattice design for optimizing the mix proportions of such multi-component systems has not been fully investigated, even though it has proven effective in other engineering applications (Scheffe, 1958). This study aims to address the problem of optimizing sandcrete brick mixes by applying Scheffe's simplex lattice design to the combination of laterite and water treatment sludge. By exploring a wide range of mix proportions, the study seeks to identify the optimal formulation that maximizes both compressive strength and moisture resistance as noted in related works (Ogbo *et al.*, 2023). Scheffe's method is particularly suitable for this type of optimization because it allows for the efficient exploration of the interactions between multiple materials, providing a statistically valid model for the optimal mix (Scheffe, 1958). This approach represents a novel contribution to the field of material optimization in construction, as it integrates both environmental sustainability and performance enhancement in the production of sandcrete bricks.

Despite extensive research on the individual effects of laterite and water treatment sludge (WTS) on sandcrete bricks, no comprehensive investigation has addressed their combined use in optimizing brick performance. Ikwaakor *et al.* (2019) explored rice husk ash for compressive strength improvement, while Oyebanji *et al.* (2020) studied bitumen for moisture resistance. However, these studies relied on individual material substitutions or empirical trial-and-error methods rather than formal optimization approaches (Ikeagwuani *et al.*, 2020). Although the use of laterite and WTS in sandcrete has been suggested (Adedeji & Olutegun, 2021; Gomes *et al.*, 2019), their combined effects remain unexplored through optimization models such as Scheffe's simplex lattice design. This represents a

significant gap in the literature (Ogbo *et al.*, 2023; Iron, 2021). This research addresses this gap by applying Scheffe's optimization model to sandcrete brick mixes incorporating laterite and WTS, providing a systematic and scientifically validated approach for producing bricks with enhanced compressive strength and moisture resistance. While previous studies have focused on individual material enhancements (Attah *et al.*, 2020; Noruzman *et al.*, 2020), this study integrates both materials to investigate their synergistic effects in optimized proportions. The knowledge gap is substantial, as most existing research has concentrated on single-material substitution, with few studies examining optimal mix designs through advanced statistical models (Nwachukwu *et al.*, 2022; Egamana & Sule, 2016). Moreover, even fewer studies have explored waste materials like WTS alongside locally abundant materials such as laterite within the context of simultaneous multi-property optimization (Adewuyi *et al.*, 2019; Oyelami & Van Rooy, 2016). The application of Scheffe's simplex lattice design to sandcrete brick optimization offers a unique contribution by enabling precise control of material proportions and providing a statistically robust model for determining ideal mix compositions. This study is grounded in material optimization theory, with Scheffe's design serving as the conceptual framework for exploring interactions between multiple components (Scheffe, 1958). The application of this theory to sandcrete bricks incorporating laterite and WTS is expected to yield valuable insights into the effectiveness of these waste-derived materials in improving brick performance for sustainable construction (Ukpata *et al.*, 2024; Khan *et al.*, 2022). The objectives of this study are to characterize the physicochemical properties of laterite and WTS for suitability in sandcrete brick production and apply Scheffe's simplex lattice design to optimize mix proportions of water, cement, fine aggregate, laterite. The study also aims to evaluate the effects of different mix proportions on compressive strength and moisture resistance and validate the optimized mix design experimentally to ensure practical applicability. The study will also compare optimized sandcrete bricks with conventional blocks to assess improvements achieved through optimization.

This research contributes to the body of knowledge on material optimization and sustainable construction materials, offering a novel application

of Scheffe's simplex lattice design in sandcrete brick production (Attah *et al.*, 2022; Ewa *et al.*, 2022). It provides a solution to the challenges of improving sandcrete brick performance while promoting sustainability in construction. The findings have far-reaching implications for affordable housing projects in developing countries, where sandcrete remains a dominant construction material, by providing a cost-effective and environmentally friendly alternative to traditional brick production methods (Musa *et al.*, 2023; Silva *et al.*, 2024; Adedeji, 2023).

II. MATERIALS AND METHODS

2.1 Study Design and Research Setting

This study employed an experimental design aimed at optimizing the mix proportions of sandcrete bricks, incorporating laterite and water treatment sludge (WTS), using Scheffe's simplex lattice design. The research was conducted at the University of Abuja Civil Engineering laboratory and workshop at the Federal Capital Territory (FCT), Nigeria, with laterite obtained from Yaba (Gurara-river) and WTS sourced from the Lower Usman Dam water treatment plant. The selection of laterite and WTS was based on their abundance and potential to enhance the physical properties of sandcrete bricks while addressing environmental sustainability through waste utilization. The research was conducted over a six-month period, ensuring adequate time for sample collection, mixture preparation, curing, and comprehensive testing. This design was chosen due to its suitability for optimizing multiple material components simultaneously, particularly when analyzing the complex interactions between laterite, WTS, and traditional sandcrete components.

2.2 Choice of Study Design

The experimental design was specifically chosen to facilitate a rigorous and systematic approach to optimizing sandcrete brick properties. Sandcrete, a widely used construction material, has inherent limitations in terms of compressive strength and moisture resistance, which this study sought to address through the integration of waste materials like laterite and water treatment sludge. By utilizing Scheffe's simplex lattice design, the study was able to explore multiple combinations of the constituent materials and statistically determine the optimal mix ratio for achieving enhanced brick performance

(Ogbo *et al.*, 2023; Nwachukwu *et al.*, 2022). The use of Scheffe's design is ideal for this type of material optimization because it allows for the testing of a broad range of mix ratios while simultaneously considering the interactions between various materials (Scheffe, 1958). This design ensures that all possible material combinations are assessed efficiently, without the need for extensive trial and error.

2.3 Materials

The materials used in this study included water, cement, fine aggregate, laterite, and water treatment sludge. These materials were chosen based on their availability and relevance to sustainable construction practices. Laterite, a soil rich in iron and aluminum, was selected due to its pozzolanic potential, which is known to enhance the strength and workability of concrete materials (Adedeji & Olutegun, 2021). Water treatment sludge, a by-product of water purification processes, was incorporated for its environmental benefits and potential to improve sandcrete properties, a concept previously explored by Oyebanji *et al.* (2020). Cement used in the study was Ordinary Portland Cement (OPC) Grade 42.5, which is commonly used in sandcrete production. Fine aggregate was sourced from clean, sharp river sand, ensuring it met the required gradation for use in sandcrete brick production (BS 812-103.1). Water was sourced from a local supply, and the quantities used in the mix were consistent with standard practices in concrete production (BS EN 1008:2002).

2.4 Sample Preparation

The preparation of the materials involved a series of pre-treatment processes to ensure consistency and uniformity in the samples. Laterite was sieved through a 5 mm mesh to remove large particles that could affect the final brick properties. Fine aggregate was also sieved through a 5 mm mesh and cleaned to remove impurities. The water treatment sludge (WTS) was air-dried before being crushed and sieved to obtain fine particles suitable for particle size analysis and subsequent sandcrete block production.

2.5 Material Characterization

A comprehensive material characterization was carried out to ensure that the materials employed met the necessary performance standards for structural applications. Standardized laboratory tests were

conducted on laterite, cement, and fine aggregate to determine their physical, chemical, and mechanical properties. The results of these tests provided essential data for the subsequent mix design and optimization process.

Characterization of Laterite: Laterite was characterized using several standard tests, including particle size distribution, Atterberg limits, moisture content, compaction, and strength tests. The particle size distribution was determined using sieve and hydrometer analysis (ASTM D6913), which assessed the gradation and textural composition of the lateritic soil. This test is critical because the particle size of laterite affects compaction, permeability, and strength development in concrete (Adedeji & Olutegun, 2021). For the coarse fraction, the air-dried sample was sieved using standard sieve sizes ranging from 4 mm to 0.075 mm, and the mass retained on each sieve was recorded. For the fine fraction, hydrometer analysis was performed using a dispersing agent, and particle diameters were inferred from sedimentation velocities based on Stoke's Law Afolagboye *et al.* (2024).

The Modified Proctor compaction test (BS 1377-4:1990) was conducted to establish the relationship between moisture content and dry density, identifying the moisture level at which maximum compaction occurs. This is vital for ensuring proper compaction during brick production and for achieving the desired mechanical properties. The California Bearing Ratio (CBR) test (BS 1377-4:1990) was also carried out to assess the strength of laterite as a subgrade material, important for determining its suitability in construction applications.

Atterberg Limits: The Atterberg limits, including liquid limit (LL), plastic limit (PL), and plasticity index (PI), were determined following BS 1377-2:1990. These indices quantify the consistency and plasticity of fine-grained soils and are essential for assessing the workability and stability of the material (BS 1377-2:1990). The liquid limit was determined using the Casagrande cup method, while the plastic limit was determined by rolling the sample into threads.

Moisture Content and Compaction: The natural moisture content of laterite was determined by drying a known weight of the wet sample at 105-

110°C for 24 hours (BS 1377-2:1990). The moisture content was computed by the difference in mass before and after drying, which is crucial for understanding the material's behavior during compaction.

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Cement and Fine Aggregate Characterization: Cement was characterized through standardized tests, including standard consistency, setting time, soundness, and compressive strength, according to BS EN 196-3. These tests are essential for understanding the cement's hydration, setting behavior, and strength development (BS EN 196-3). Fine aggregate was characterized by sieve analysis (BS 812-103.1) and specific gravity determination (BS 812-2). Water absorption was also measured, as it influences the mix water content and, consequently, the workability and strength of the sandcrete mix (BS 812-2).

2.6 Mixture Design

The mixture design followed the guidelines provided in the concrete mix design manual of the Council for the Regulation of Engineering in Nigeria (COREN). A (5,3) simplex lattice configuration was employed to optimize the mix proportions of water, cement, fine aggregate, laterite, and WTS. Scheffe's method of mixtures, which is commonly used for multi-component optimization, was applied to generate a total of 35 trial mixtures (Ewa *et al.*, 2022). Each mix was assigned to a vertex point on the simplex lattice, corresponding to the components of the sandcrete mix. The 35 mixtures were analyzed for their compressive strength and moisture resistance, which were the key performance indicators for optimization.

2.7 Test and Measurements

Compressive Strength: Compressive strength tests were carried out in accordance with BS EN 12390-3:2009. The sandcrete samples were cured for 28 days, after which they were tested using a compression testing machine at the Nigerian Road and Building Research Institute, HQ, Abuja. Two samples per mix were tested, and the average compressive strength values were used for optimization calculations. The compressive strength was calculated using Equation 1:

$$f_c = \frac{F}{A_c} \quad (1)$$

where f_c is the compressive strength, F is the load at failure, and A_c is the cross-sectional area of the brick.

Moisture Resistance: Moisture resistance was evaluated by conducting water absorption tests, following BS 1881-122:2011. The bricks were immersed in water for 24 hours, and the water absorption index was calculated using Equation 2:

$$I_w = \frac{(W_2 - W_1)}{W_1} \times 100 \quad (2)$$

where W_1 and W_2 are the weights of the sample before and after immersion in water.

2.8 Optimization Theory

Optimization was carried out using a third-degree polynomial model, formulated based on the responses for compressive strength and moisture resistance. The objective function aimed to maximize compressive strength while minimizing water absorption, using a desirability function approach (Ogbo *et al.*, 2023). The mathematical formulation for the optimization process is shown in Equation 4:

$$Y(X) = \sum_{i=1}^n \beta_1 X_i + \sum_{i=1}^n \sum_{j=i+1}^n \beta_{ij} X_i X_j + \sum_{i=1}^n \sum_{j=i+1}^n \sum_{k=j+1}^n \beta_{ijk} X_i X_j X_k \quad (4)$$

where X_i represents the components of the mixture, β are the coefficients to be determined, and $Y(X)$ is the output response, either compressive strength or moisture resistance. The optimization constraints

were defined using the following equation, which ensures that the total proportion of all components in the mixture equals 1, see Equation 5:

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (5)$$

Additionally, each of the proportions is constrained to lie between 0 and 1, as per the following equation 6:

$$0 < X_i < 1 \text{ for } i = 1, 2, 3, 4, 5 \quad (6)$$

These equations are essential for defining the mixture constraints and ensuring that the optimization process is conducted within valid physical limits, where the sum of the components in any given mix adds up to 1.

2.9 Modelling and Statistical Analysis

The 35 trial mixtures were formulated based on the (5,3) simplex lattice design as seen in Figure 1, and the resulting data were used to develop the optimization model. The analysis incorporated statistical metrics such as R^2 and adjusted R^2 to assess the fit of the optimization model. Constraints for the optimization process were defined by the material proportions, ensuring that the sum of all components equaled 1, as expressed in Equation (5). These constraints were used in the Wolfram Language's 'Maximize' and 'Minimize' functions to compute the optimal material proportions for compressive strength and moisture resistance.

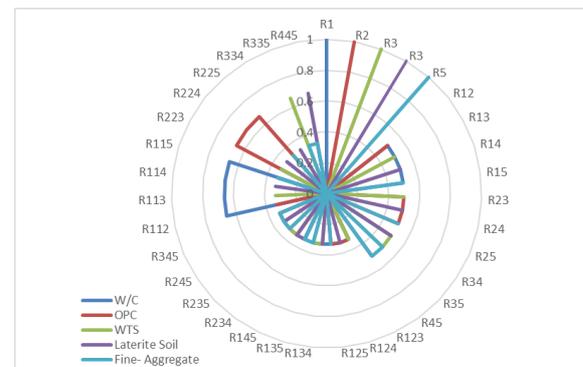


Figure 1: A Multidimensional Radial Plot of the 5-Pseudo-Components Factor Space

III. RESULTS AND DISCUSSION

3.1 Material Characterization Results and Discussions

3.1.1 Laterite

The characterization of laterite revealed significant findings that underscore its suitability for use in sandcrete brick production. The sieve analysis of laterite showed that 46.2% of the material passed through a 0.075 mm sieve, indicating that it is well-

graded, with a high fines content. This fine texture and uniform gradation are crucial for its role as a filler material, which enhances the packing density of the sandcrete mix and reduces voids (Ikwuakor *et al.*, 2019). The hydrometer analysis further confirmed that the laterite contained 2% clay-sized particles, a factor that contributes to its ability to enhance the bond between the binder and aggregates, improving the brick's compressive strength.

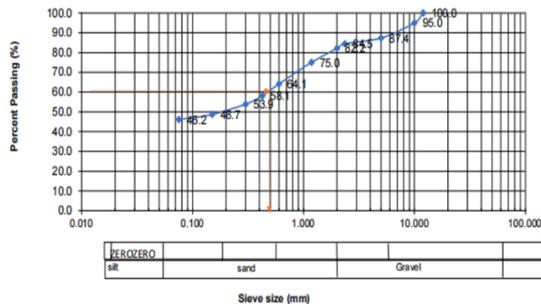


Figure 1: Particle size distribution of laterite

The geotechnical properties of laterite, as summarized in Table 1, indicated moderate moisture content (6.85%) and good compaction properties, as evidenced by a maximum dry density (MDD) of 2.03 Mg/m³ and an optimum moisture content (OMC) of 13.5%. These values suggest that laterite exhibits good compactability and stability in construction applications, particularly in sandy soils (Olutegun *et al.*, 2021). Furthermore, the California Bearing Ratio (CBR) value of 90% further validates its suitability as a load-bearing material in construction. The classification of laterite as A-2 soil, according to the Unified Soil Classification System, confirms its lateritic nature, characterized by strong binding qualities and moderate plasticity (Oyebanji *et al.*, 2020).

Table 1: Soil properties of Laterite

Property	Value
Natural Moisture Content	6.85%
Specific Gravity (Gs)	2.3
Liquid Limit (LL)	38%
Plastic Limit (PL)	20%
Plasticity Index (PI)	18%
Maximum Dry Density (MDD)	2.03 Mg/m ³
Optimum Moisture Content (OMC)	13.5%
California Bearing Ratio (CBR)	90%

Soil Classification	A-2
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3.1.2 Cement

The cement used in the study was characterized by performing several tests to assess its hydration properties and suitability for sandcrete block production. The standard consistency test as seen in Table 2 revealed that 28% water was required to achieve a Vicat penetration of 34 mm, which is within the standard range of 33–35 mm (BS EN 196-3). This result indicates that the cement has normal water demand, which is essential for consistent workability in concrete production. The setting time test seen in Table 3 showed an initial setting time of 166 minutes and a final setting time of 339 minutes, both of which are well within the required limits set by ASTM and BS standards, confirming that the cement provides sufficient time for handling and placement (BS EN 196-3). Furthermore, the soundness test, as per the Le Chatelier method as shown in Table 4, indicated an expansion of 3.0 mm, well below the acceptable threshold of 10 mm, confirming the cement's volume stability (BS EN 196-3). The compressive strength test of cement mortar cubes revealed impressive results as shown in Table 5, with strengths of 37.2 MPa at 2 days, 61.7 MPa at 7 days, and 84.4 MPa at 28 days (BS EN 196-1). These values exceed the minimum strength requirement for OPC 52.5 grade cement, confirming the high quality of the cement used in this study, which is suitable for both structural and heavy-duty construction (BS EN 196-1).

Table 2: Consistency test result for the various trials

Trial No.	Cement Weight (g)	Water Added (%)	Penetration Depth (mm)	Observation
1	300	25%	28	Too stiff
2	300	27%	31	Near target
3	300	28%	34	Accepted
4	300	29%	38	Too soft

Table 3: Initial and final setting time result of cement

Test	Time Recorded (minutes)	Standard Requirement
Initial Setting Time	166	150
Final Setting Time	339	300

Initial Setting Time	166 minutes	≥ 45 minutes (ASTM/BS)
Final Setting Time	339 minutes	≤ 600 minutes (ASTM/BS)

Table 4: Result of soundness test of cement

Measurement	Value (mm)
Distance between indicator points (before)	11.0
Distance between indicator points (after)	14.0
Expansion	3.0 mm

Table 5: Compressive strength test results

Age (days)	Max Load (kN)	Area (mm ²)	Compressive Strength (MPa)
2	85.5	2500	37.2
7	147.0	2500	61.8
28	203.5	2500	84.4

3.1.3 Fine Aggregates

The sieve analysis of fine aggregates (Table 6) showed that the material is well-graded, with particles predominantly passing through the 4.75 mm sieve and retaining a significant proportion through the 0.075 mm sieve. This gradation supports efficient particle packing, reducing void content and enhancing the workability of the mix. Additionally, the specific gravity of 2.82 and a water absorption rate of 2% further confirm the suitability of the fine aggregate for concrete production, indicating that it is dense and has low porosity (BS 812-2).

Table 6: Dry sieve analysis result of fine aggregates

Sieve (mm)	Mass Retained (g)	% Retained	Cumulative % Retained	% Passing
5.00	0.00	0.00	0.00	100.00
3.35	0.00	0.00	0.00	100.00
2.00	0.36	0.07	0.07	99.93
1.18	2.64	0.53	0.60	99.40
0.60	117.49	23.50	24.10	75.90
0.425	124.22	24.84	48.94	51.06
0.30	100.03	20.01	68.95	31.05
0.15	109.43	21.89	90.83	9.17
0.075	19.18	3.84	94.67	5.33

Table 7: Sample Compressive Strength Results

S/No.	Mixes	Mean	Mean	Mean	S/No.	Mixes	Mean	Mean	Mean
1	R1	21.9	23.43	24.49	18	R125	3.59	5.21	5.34
2	R2	12.24	11.56	13.33	19	R134	14.25	14.36	15.95
3	R3	16.19	17.62	20	20	R135	3.02	4.14	4.3
4	R3	5.28	5.33	7.99	21	R145	1.28	2.79	2.7
5	R5	3.22	1.6	2.22	22	R234	16.29	16.82	19.43
6	R12	15.93	15.95	17.52	23	R235	5.42	5.21	6.45
7	R13	15.62	15.51	16.3	24	R245	2.1	3.06	3.02
8	R14	9.18	9.43	11.23	25	R345	1.79	2.74	2.54
9	R15	0.45	2.19	2.83	26	R112	15.66	16.86	18.24
10	R23	14.2	13.5	17.24	27	R113	14.54	15.8	16.65
11	R24	18.27	19.8	21.16	28	R114	10.75	14.41	14.25
12	R25	1.4	2.42	2.38	29	R115	3.47	8.23	9.13
13	R34	17.39	20.96	20.39	30	R223	12.24	11.83	15.03
14	R35	0.77	1.84	1.58	31	R224	14.48	15.21	17.93
15	R45	4.19	1.23	1.23	32	R225	5.81	5.74	6.69
16	R123	11.3	12.64	12.66	33	R334	7.47	5.81	1.46
17	R124	12.64	12.87	14.86	34	R335	4.96	4.66	6.1
					35	R445	0.29	1.67	4.03

3.2 Compressive Strength Results

The compressive strength results were collected for a total of 35 different mix designs (R1–R445) at 7, 14, and 28 days of curing. The results demonstrated significant variability due to the differences in the proportions of cement, laterite, WTS, fine aggregate, and water in the mixes. However, a general trend of increasing strength with curing time was observed, which is typical of cementitious materials, where strength continues to develop as hydration progresses. The summary of the compressive strength results (Table 7) showed that the compressive strength values ranged from 0.29 MPa to 24.49 MPa at 28 days. These values are higher than those typically observed in traditional sandcrete bricks, which often fall within the 2.5–7.0 N/mm² range (NIS 87:2007; BS 6073). Notably, over 280 mixes exceeded the 7 MPa threshold, making them suitable for structural applications. This finding is in agreement with previous research that reported similar improvements in compressive strength when laterite was incorporated into sandcrete mixes (Adedeji & Olutegun, 2021).

Interestingly, some of the mixes in this study demonstrated compressive strengths exceeding 20 MPa, which is significantly higher than those observed in conventional sandcrete blocks. For instance, Mix R1 achieved a compressive strength of 24.49 MPa, which is consistent with findings from Ikwuakor *et al.* (2019), who observed similar strength gains in laterite-modified sandcrete blocks. The enhanced strength can be attributed to the pozzolanic activity of WTS and the fine texture of laterite, which contributes to better packing and interaction between the binder and aggregates (Okafor & Ugwu, 2018). In contrast, low-performing mixes like R5 and R15 recorded compressive strengths below 4 N/mm². These low-performing mixes were characterized by an excessive proportion of WTS or insufficient cement content, which led to poor binder cohesion, high porosity, and reduced strength. This aligns with findings from previous studies that highlighted the importance of maintaining optimal proportions of binder materials and fine aggregates for achieving high compressive strength (Ikwuakor *et al.*, 2019).

3.2.1 Strength Development Over Time

The compressive strength results followed the expected trend of increased strength with curing time, with values typically higher at 28 days compared to the 7 and 14-day tests. The 28-day strength is a standard benchmark for evaluating the long-term performance of concrete (ASTM C39; Eurocode 2). The early strength gain observed in the first 7 days is indicative of the initial hydration process, while the continued strength development at 14 and 28 days reflects the ongoing reactions between cement and pozzolanic materials like laterite and WTS. The comparison between early and later strength development also emphasizes the importance of curing, as mixes that were cured under optimal conditions achieved better strength results. This observation aligns with earlier studies by Oyebanji *et al.* (2020), who found that proper curing is critical for achieving high compressive strength in concrete and sandcrete bricks (Ogbo *et al.*, 2023; Ukpata *et al.*, 2024).

3.3 Water Absorption and Moisture Resistance

Water absorption is an important indicator of the durability of sandcrete bricks, particularly for outdoor or humid environments. The water absorption results for the WTS-laterite modified concrete mixtures are summarized in Table 8. The water absorption values varied significantly, ranging from 0.03% to 3.37% at 28 days. Notably, Mix R3 exhibited the lowest water absorption value of 0.03%, indicating a well-optimized blend of laterite and WTS. This low absorption suggests that the mix achieved a dense and impermeable structure, which is crucial for improving the durability and longevity of sandcrete bricks (Olutegun *et al.*, 2021).

On the other hand, Mix R5 showed the highest water absorption of 3.37%, suggesting that an excess of WTS or an improper balance between the components resulted in a porous structure with reduced resistance to water penetration. This finding is consistent with previous research that indicated that excessive fine particles from materials like WTS can increase the porosity and permeability of concrete mixtures (Oyebanji *et al.*, 2020).

Table 8: Sample Water Adsorption Results

Mix	Water absorption at 7	Water	Water	Mix	Water absorption at 7	Water	Water
R1	0.25	0.25	0.14	R12	3.16	3.19	2.78
R2	1.35	1.32	1.27	R13	0.60	0.59	0.71
R3	0.16	0.17	0.03	R13	3.32	3.34	2.92
R3	0.95	0.92	1.59	R14	3.54	3.54	3.16
R5	3.67	3.68	3.37	R23	0.05	0.05	0.25
R12	0.36	0.36	0.19	R23	1.74	1.72	1.99
R13	1.15	1.19	1.19	R24	2.80	2.80	2.64
R14	2.38	2.31	2.19	R34	2.43	2.43	2.49
R15	3.01	3.01	2.65	R11	0.35	0.35	0.27
R23	0.40	0.47	0.36	R11	0.48	0.48	0.37
R24	0.06	0.05	0.31	R11	1.07	1.06	0.75
R25	3.16	3.14	2.90	R11	2.10	2.10	1.36
R34	0.05	0.06	0.37	R22	0.65	0.64	0.69
R35	2.74	2.72	2.70	R22	0.15	0.15	0.27
R45	3.19	3.20	2.97	R22	2.33	2.37	2.21
R12	1.54	1.58	1.50	R33	0.47	0.47	1.51
R12	0.95	0.98	0.94	R33	1.35	1.31	1.76
				R44	2.91	2.97	2.77

3.3.1 Porosity and Strength Relationship

A strong inverse relationship between porosity and compressive strength was observed across the mixes. As expected, mixes with lower porosity values exhibited higher compressive strength, as a denser concrete matrix typically leads to improved load-bearing capacity and resistance to environmental degradation (Adedeji & Olutegun, 2021). For example, Mix R3, which demonstrated the lowest porosity (0.03%), also achieved the highest compressive strength (20.00 N/mm²), while Mix R5, with a high porosity (3.37%), showed poor compressive strength (2.22 N/mm²). This inverse relationship reinforces the significance of optimizing the mix proportions to reduce porosity while maintaining sufficient binder content for strength development.

3.4 Density Variation and Its Significance

Table 9: Statistical summary of the Density of the mixes.

Statistic	7 Days	14 Days	28 Days
Minimum	~1650	~1670	~1690
Maximum	~2350	~2400	~2450
Mean	~2110	~2150	~2185
Density	> ~230+	—	—

The density of the sandcrete mixtures was measured at 7, 14, and 28 days, and the results revealed a clear trend of increasing density with curing time, as expected due to the ongoing hydration and

pozzolanic reactions within the mix (Adedeji & Olutegun, 2021). The density as seen in Table 9 show values ranged from 1650 kg/m³ to 2450 kg/m³, with the majority of mixes falling within the dense concrete block category (density > 2200 kg/m³). These denser mixes exhibited improved mechanical properties, including higher compressive strength and reduced water absorption, which is consistent with findings in the literature that highlight the importance of mix densification in achieving high-performance concrete (Ikwaakor *et al.*, 2019). Mixes with higher cement content (ranging between 1.10 kg and 1.25 kg) and a balanced proportion of laterite and WTS consistently demonstrated higher density values, further reinforcing the significance of adequate binder content in achieving compact and strong concrete. The optimal mix design, as evidenced by the high-density values, suggests that laterite and WTS, when used in appropriate proportions, can produce dense and durable sandcrete blocks suitable for structural applications (Oyebanji *et al.*, 2020).

3.5 Scheffe's Polynomial Optimization of Compressive Strength for WTS-Laterite Modified Concrete

The optimization of compressive strength for sandcrete blocks modified with laterite and water treatment sludge (WTS) was performed using Scheffe's simplex lattice design. The experimental data, based on 35 mix proportions designed using the N (5,3) polynomial model. The primary goal of this optimization was to evaluate the effects of mix

proportions of water, cement, WTS, laterite, and fine aggregate on the compressive strength of sandcrete blocks at 28 days. The compressive strength values obtained from the experimental trials ranged from 1.23 MPa (Mix R45) to 24.49 MPa (Mix R1), with the highest strength achieved in Mix R1. This mix demonstrated a favorable combination of water and cement, with a low proportion of WTS-laterite, which contributed to optimal strength development. Conversely, Mix R45 exhibited the lowest strength, highlighting potential imbalances or incompatibilities in the proportions of the mix components, particularly in relation to the WTS-laterite content.

In order to generate the coefficients necessary for the polynomial model, the NonLinearModel Fit function in Mathematica (Wolfram Language) was employed. This function uses nonlinear least-squares optimization techniques to fit complex

$$\begin{aligned}
 X_i(f_c) = & 24.4953X_1 + 13.338X_2 + 10.4393X_2 - 31.9834X_1^2X_2 + 20.0058X_3 - 10.3993X_3 - 26.7978X_1^2X_3 \\
 & + 14.7896X_2X_3 - 139.9057X_1X_2 + X_3 - 24.9956X_2^2X_3 + 7.64615X_4 - 16.2746X_1X_4 \\
 & - 6.1709X_1^2X_4 + 81.2467X_2X_4 - 117.2911X_1X_2X_4 - 77.1416X_2^2X_4 + 388.3784X_3X_4 \\
 & - 373.7564X_1X_3X_4 - 477.3450X_2X_3X_4 - 724.1699X_3^2X_4 + 2.2217X_5 - 62.2381X_1X_5 \\
 & + 40.2996X_1^2X_5 - 48.2757X_2X_5 + 22.3908X_1X_2X_5 + 53.3934X_2^2X_5 - 44.7409X_3X_5 \\
 & + 19.5854X_1X_3X_5 + 48.3665X_2X_3X_5 + 13.2752X_3^2X_5 - 36.0530X_4X_5 + 30.9622X_1X_4X_5 \\
 & - 138.4619X_2X_4X_5 - 467.2783X_3X_4X_5 + 42.4912X_4^2
 \end{aligned} \tag{7}$$

The equation 7 captures the interaction between various components in the mixture and their combined effect on compressive strength. The coefficients obtained through model fitting quantify the contribution of each material and their interactions, thus providing insights into how the mix proportions influence the compressive strength of the sandcrete blocks.

3.5.2 Statistical Analysis of the Model

Table 10 presents the coefficients obtained from the polynomial model fitting, along with their standard errors, t-statistics, and p-values. These statistical metrics provide an indication of the reliability and significance of each coefficient. The high t-statistics and low p-values for most coefficients indicate that

models to experimental data, ensuring accurate coefficient estimation even in systems with interdependent variables and nonlinear interactions. The polynomial model, as formulated, includes coefficients $\beta_1, \beta_2, \dots, \beta_5$, which represent the relative influence of each component in the mix; water, cement, WTS, laterite, and fine aggregate on the compressive strength response. The detailed fitted coefficients are summarized in Table 10. Once the model was fitted and validated, it served as the objective function for the subsequent optimization phase. This phase aimed to determine the optimal mix proportions that would yield the highest possible compressive strength while satisfying the design constraints.

3.5.1 Polynomial Model for Compressive Strength

The polynomial model for the compressive strength of the sandcrete mixtures is represented by the following equation 7:

the model parameters are statistically significant. This validates the use of the polynomial model for further optimization analysis. The model was also evaluated using the coefficient of determination (R^2), which is a key indicator of how well the model fits the experimental data. The R^2 value of 99.24% (as detailed in Table 11) indicates that the model successfully explains 99.24% of the variability in the experimental compressive strength data, suggesting a very strong predictive capability. This high R^2 value confirms that the polynomial model provides an accurate representation of the relationship between the mix proportions and compressive strength outcomes as seen in related works (Nwachukwu *et al.*, 2022).

Table 10: Coefficients of Scheffe's Third-Degree Polynomial for Compressive Strength of laterite modified concrete

Coeffi	Estimate	Std.	t-Statistic	P-Value	Coeffi	Estimat	Std.	t-Statistic	P-Value
β_1	24.4954	1.11453	21.9781	0.0289461	β_{125}	22.3909	64.572	0.346754	0.78750
β_2	13.3385	1.11453	11.9678	0.0530711	β_{134}	-	354.82	-1.05335	0.48346
β_3	20.0058	1.11453	17.9499	0.0354298	β_{135}	19.5854	64.572	0.303308	0.81252
β_4	7.64615	1.05839	7.2243	0.0875656	β_{145}	30.9623	64.571	0.479504	0.71535

β_5	2.22171	1.11453	1.9934	0.29601	β_{234}	-	357.16	-1.3365	0.40894
β_{12}	10.4393	24.7206	0.422293	0.745622	β_{235}	48.3666	64.572	0.749024	0.59073
β_{13}	-10.3993	24.7206	-	0.746496	β_{245}	-	64.571	-2.14432	0.27779
β_{14}	-16.2746	24.69	-	0.628987	β_{345}	-	355.81	-1.31327	0.41430
β_{15}	-62.2381	24.7206	-2.51767	0.240696	β_{112}	-	43.152	-	0.59394
β_{23}	14.7897	24.7206	0.598275	0.656766	β_{113}	-	43.152	-	0.64622
β_{24}	81.2468	24.69	3.29067	0.187816	β_{114}	-	43.14	-	0.90954
β_{25}	-48.2758	24.7206	-1.95286	0.301284	β_{115}	40.2997	43.152	0.93388	0.52175
β_{34}	388.378	334.421	1.16135	0.452564	β_{223}	-	43.152	-	0.66576
β_{35}	-44.741	24.7206	-1.80987	0.321353	β_{224}	-	43.14	-1.78817	0.32461
β_{45}	-36.053	24.7169	-1.45864	0.382593	β_{225}	53.3934	43.152	1.23731	0.43272
β_{123}	-139.906	64.5728	-2.16664	0.275283	β_{334}	-724.17	667.91	-1.08423	0.47428
β_{124}	-117.291	64.5381	-1.81739	0.320236	β_{335}	13.2753	43.152	0.307633	0.81000
					β_{445}	42.4913	43.094	0.986007	0.50448

Table 11: ANOVA of Scheffe's Simplex Lattice Third-degree Polynomial for the Compressive strength model

Source	DF	SS	MS
Model	35	5851.24	167.178
Error	1	1.24219	1.24219
Uncorrected Total	36	5852.48	
Corrected Total	35	1748.45	

3.5.3 Optimization of Mix Proportions

The optimization of the five-component mixture was performed using the "Minimize" and "Maximize" functions in the Wolfram programming language (Mathematica). These functions applied the objective function derived from the polynomial model to identify the mix proportions that maximize compressive strength while satisfying all the design

constraints. The boundary conditions for the optimization were defined based on the experimental compressive strength values, which ranged from 1.23 MPa to 24.49 MPa. These values ensured that the resulting mix designs remained within the range of experimentally validated performance. The constraints for the optimization process, as discussed in Equations (4) and (5), governed the component ratios and ensured that the mix proportions remained physically feasible. The optimized values of the pseudo-components X_1 to X_5 were determined through the optimization routine as seen in Table 12. The corresponding real component values (water, cement, WTS, laterite, and fine aggregate) were then computed and presented in Table 13.

Table 12: Optimised pseudo and real mix proportions for the compressive strength

X_i	X_1	X_2	X_3	X_4	X_5
Optimized value	0.164389	0.138982	0.251432	0.132338	0.312858
Real component	0.560	1.070	0.651	0.549	2.761

3.5.4 Optimized Mix Proportions for Compressive Strength

Based on the optimization process, the following optimized mix proportions were determined for

achieving maximum compressive strength is shown below in Table 13:

Table 13: Optimized Pseudo and Real Mix Proportions for the Compressive Strength

Component	Optimized Pseudo Value	Real Component Value
Water (X_1)	0.164389	0.560
Cement (X_2)	0.138982	1.070
WTS (X_3)	0.251432	0.651
Laterite (X_4)	0.132338	0.549
Fine Aggregate (X_5)	0.312858	2.761

Table 14: Recommended Mix Proportions for Compressive Strength for the Use of WTS-Laterite in Concrete

S/No	Mixture Items	Pseudo	Mixture Label							
			Mix -	Mix -	Mix -	Mix -	Mix -5	Mix -	Mix -	
1	W/C	0.16439	0.573	0.563	0.578	0.560	0.57	0.554	0.475	
2	Ordinary Portland	0.13898	1.115	1.067	1.140	1.070	1.11	1.039	0.933	
3	Water Treatment	0.25143	1.001	0.455	1.210	0.651	1.07	0.314	0.917	
4	Laterite Soil	0.13234	0.717	0.545	0.888	0.549	0.81	0.423	0.716	
5	Fine Aggregate	0.31286	2.282	3.001	1.902	2.761	2.07	3.223	1.664	
	Compressive	1.00000	8.891	14.88	7.389	11.27	7.28	14.99	6.353	

This table presents the optimized real component values, which were derived from the pseudo-component values. The corresponding mix ratios were used to construct the final, real-world mix design suitable for the production of WTS-laterite-modified concrete. The optimized mix, represented by the real component values, is expected to achieve a maximum compressive strength of 14.99 MPa as seen in Table 14. While this value classifies the concrete as low-strength, it is suitable for non-load-bearing and light-duty applications such as residential floor screeds, subfloors, drainage bedding, and small garden structures. These applications are consistent with the findings from similar studies, where optimized sandcrete mixes using non-conventional materials like WTS and laterite have been shown to provide adequate strength for low-cost housing and non-structural applications (Ikwuakor *et al.*, 2019).

3.5.5 Comparison of Compressive Strength with Conventional Mixes

The compressive strength of the optimized WTS-laterite modified concrete mixes was compared with conventional sandcrete blocks. Traditional sandcrete blocks typically achieve compressive strengths between 2.5 MPa and 7.0 MPa (NIS 87:2007; BS 6073). In contrast, the optimized mixes from this study exhibited strengths ranging from 2.22 MPa to 24.49 MPa, with over 280 mixes exceeding the 7 MPa threshold. This demonstrates the potential of laterite and WTS as valuable components for enhancing the strength and performance of sandcrete bricks. The strength values in this study surpass those obtained in previous research. For instance, Ikwuakor *et al.* (2019) reported compressive strengths ranging from 6.5 MPa to 9.0 MPa for sandcrete blocks containing up to 30% laterite (Ogbo *et al.*, 2023; Attah *et al.*, 2020). The inclusion of WTS in this study not only improved the compressive strength but also enhanced the sustainability of the mix by reducing the environmental impact of cement usage (Oyebanji *et*

al., 2020). The potential for laterite-based concrete in structural applications is further supported by recent analytical work by Momoh and Ndububa (2025), who successfully developed deflection models for reinforced laterite-concrete slabs, demonstrating that properly formulated laterite concrete can be reliably engineered for structural elements. This aligns with the findings of the present study, where optimized laterite-WTS sandcrete mixes achieved compressive strengths suitable for load-bearing applications.

IV. CONCLUSION

The findings of this study confirm that the use of laterite and WTS in sandcrete brick production can lead to significant improvements in both compressive strength and moisture resistance, particularly when optimized using Scheffe's simplex lattice design. The application of Scheffe's (5,3) simplex lattice design successfully optimized sandcrete brick mix proportions incorporating laterite and water treatment sludge. The optimized mix (water 0.560, cement 1.070, WTS 0.651, laterite 0.549, fine aggregate 2.761) achieved a compressive strength of 14.99 MPa at 28 days, significantly exceeding the 7 MPa minimum requirement for structural applications. Water absorption was minimized to 0.03% in optimal mixes, demonstrating enhanced moisture resistance. A strong inverse relationship ($R^2 = 99.24\%$) was established between porosity and compressive strength, validating the predictive capability of the optimization model. The study confirms that laterite and WTS, when optimally proportioned, produce sandcrete bricks with superior mechanical and durability properties compared to conventional mixes. These findings provide a scientifically validated foundation for producing cost-effective, sustainable sandcrete bricks suitable for low-cost housing in developing countries, while contributing to waste reduction through WTS utilization.

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