

# Optimization of Atterberg Limits And Structural Reliability of Cement–Water Treatment Sludge Stabilized Abuja Laterite For Structural Earth Block Production

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**Abstract** — *This study examines the optimization of Atterberg limits and structural reliability of lateritic soil stabilized with Ordinary Portland Cement (OPC) and Water Treatment Sludge (WTS) for structural earth block production in Abuja, Nigeria. The research aimed to characterize the materials mineralogically, establish mixture compositions using Scheffé's polynomial model, evaluate Atterberg limits and 28-day compressive strength, and determine the optimal mix ratio. Laboratory analyses showed that the laterite contained a combined pozzolanic oxide content ( $\text{SiO}_2 + \text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$ ) of 76.22%, meeting pozzolanic standards, while WTS recorded 68.93% with notable sulphate content (0.97%), indicating limited suitability as a standalone binder. The raw laterite exhibited a low plasticity index (0.42%), while stabilization with OPC and WTS improved shrinkage resistance and moisture stability. Compressive strength tests revealed that the control sample attained 3.508 MPa, whereas optimized mixes B12 and D12 achieved 3.469 MPa (98.9% of control) with enhanced dimensional stability. Scheffé's optimization yielded coefficients of determination ( $R^2$ ) above 99%, confirming strong model reliability. The optimal mix composition was approximately 0.39 OPC, 0.08 WTS, 0.53 water, and trace laterite. The study concludes that controlled incorporation of WTS ( $\leq 8\%$ ) with OPC effectively stabilizes lateritic soil for durable, sustainable, and low-cost earth block production, demonstrating Scheffé's model as a robust optimization tool for material design.*

**Keywords** — *Lateritic soil; Water treatment sludge (WTS); Ordinary Portland cement (OPC); Atterberg limits; Compressive strength; Scheffé's mixture design; Structural earth blocks; Sustainable construction materials.*

## I. INTRODUCTION

Lateritic soils are widely distributed across tropical regions and constitute one of the most accessible construction materials for low-cost housing and infrastructural development. In Nigeria, Abuja

laterite is extensively utilized for masonry units and road construction due to its availability and relatively favorable engineering properties. However, the geotechnical performance of untreated lateritic soils is often governed by their plasticity characteristics and moisture susceptibility, which significantly influence strength, durability, and structural reliability. The Atterberg limits—Liquid Limit (LL), Plastic Limit (PL), and Plasticity Index (PI)—are critical parameters for evaluating soil consistency and predicting field behavior under varying moisture conditions (Das & Sobhan, 2014; Lambe & Whitman, 1969). Previous studies have demonstrated strong correlations between Atterberg limits, compaction characteristics, and load-bearing capacity of fine-grained soils (Terzaghi et al., 1996). High plasticity indices are generally associated with excessive deformation, shrinkage cracking, and reduced structural efficiency, while optimized plasticity enhances dimensional stability and long-term performance. Therefore, modifying the consistency characteristics of lateritic soils is essential for improving their suitability in structural earth block production. Cement stabilization remains one of the most effective methods for enhancing soil strength and durability. Ordinary Portland Cement (OPC) improves mechanical properties through hydration reactions that produce calcium silicate hydrate (C–S–H) gel, forming a rigid bonding matrix within the soil structure (Das, 2013; Neville, 2011). The stabilization of lateritic soils with cement has been shown to significantly improve compressive strength and reduce plasticity, making them suitable for load-bearing applications (Das, 2013). However, the environmental implications of OPC production—particularly its high carbon footprint—have motivated research into supplementary cementitious materials capable of partially replacing cement while maintaining performance (Olivier et al., 2016).

Agricultural and industrial by-products such as rice husk ash (RHA) have been widely investigated as partial cement replacements due to their pozzolanic properties (Ganesan et al., 2008; Zhang & Malhotra, 1996). Optimization studies using Scheffé's mixture theory have successfully demonstrated that cement–RHA blends can achieve desirable mechanical performance when proportioned appropriately (Scheffé, 1958). These findings underscore the importance of systematic mixture modeling for achieving balanced performance in composite materials.

Water Treatment Sludge (WTS), a by-product of municipal water purification processes, has recently attracted attention as a sustainable additive in construction materials. Studies have shown that WTS can be valorized in cementitious systems and ceramic materials, contributing to improved environmental efficiency (Frias et al., 2011; Monzó et al., 2003; Vouk et al., 2014). Incorporation of sludge-based materials in construction not only diverts waste from landfills but also supports circular economy strategies in infrastructure development (United Nations Environment Programme [UNEP], 2019). Nevertheless, the influence of WTS on the plasticity and structural reliability of stabilized lateritic earth blocks remains insufficiently explored.

In structural applications such as stabilized earth blocks, compressive strength at 28 days is a primary indicator of load-bearing capacity and service performance (ASTM International, 2017; British Standards Institution, 2011). However, strength alone does not ensure durability; the interaction between consistency limits, moisture sensitivity, and strength development must be evaluated simultaneously. Consequently, a multi-response optimization framework becomes necessary to balance plasticity reduction and strength enhancement.

Scheffé's mixture design method provides a robust statistical approach for modeling interaction effects among composite materials (Scheffé, 1958). By employing higher-degree polynomial formulations, the method enables prediction and optimization of material responses within defined mixture constraints. Such techniques have been effectively applied in cement-based systems to determine optimal blend compositions that satisfy both mechanical and serviceability requirements.

Despite extensive research on cement stabilization and waste-derived additives, limited studies have specifically investigated the combined optimization of Atterberg limits and structural reliability of OPC–WTS stabilized Abuja laterite for structural earth block production. Furthermore, few investigations have integrated mineralogical characterization, mechanical testing, and advanced mixture modeling within a unified optimization framework.

Therefore, this study aims to optimize the Atterberg limits and structural reliability of cement–water treatment sludge stabilized Abuja laterite using Scheffé's fourth-degree polynomial model. The research evaluates mineralogical composition, consistency parameters, and 28-day compressive strength to determine an optimal mixture proportion suitable for sustainable and structurally reliable earth block production in tropical environments.

## II. MATERIALS AND METHODS

### 2.1 Materials

#### 2.1.1 Lateritic Soil

Lateritic soil was obtained from a road construction excavation site along Sani Abacha Way, Abuja, Nigeria (Federal Capital Territory). The soil was collected at a depth of approximately 1.5 m to minimize organic contamination. Disturbed samples were transported in sealed polyethylene bags to prevent moisture loss prior to laboratory testing.

The air-dried soil was pulverized and passed through a 425 µm sieve in accordance with BS 1377-2:1990 to remove coarse fractions and ensure uniformity for consistency tests and brick production. Preliminary classification tests were conducted to determine particle size distribution and Atterberg limits.

#### 2.1.2 Ordinary Portland Cement (OPC)

Commercial-grade Ordinary Portland Cement (OPC) conforming to ASTM C150 Type I specifications was procured from a local supplier in Abuja. The cement was stored in airtight containers to prevent moisture ingress prior to mixing. OPC served as the primary stabilizing binder in the soil mixtures.

#### 2.1.3 Water Treatment Sludge (WTS)

Water Treatment Sludge (WTS) was collected from the Lower Usama Dam water treatment facility, Abuja. The sludge was air-dried for 7 days and subsequently oven-dried at  $105 \pm 5$  °C to constant mass to eliminate residual moisture. The dried material was pulverized and sieved through a 200  $\mu$ m sieve to obtain a fine powder suitable for blending with soil and cement.



Fig. 1: Sample Water Treatment Sludge (WTS)

The chemical composition of the laterite and WTS was determined using X-ray fluorescence (XRF) analysis to evaluate the combined oxide content ( $\text{SiO}_2 + \text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$ ), which is indicative of potential pozzolanic activity.



Fig. 2: Laterite Sample Before Sieving

## 2.2 Experimental Design and Mixture Modelling

A four-component mixture system consisting of laterite (L), OPC (C), WTS (S), and water (W) was investigated. Scheffé's simplex lattice mixture design of fourth degree was adopted to model the interaction effects among components.

For a mixture polynomial of degree  $m$  with  $q$  components, the number of required experimental runs ( $N$ ) is expressed as:

$$N(q, m) = \frac{(q + m - 1)!}{m! (q - 1)!} \quad (\text{Eqn 1})$$

Where  $q$  is the degree of the polynomial and  $m$  is the number of components. For this study,  $N(4,4)$  was determined to be 35 trials.

## 2.3 Sample Preparation and Brick Production

The predetermined mixture proportions were weighed using a digital balance with  $\pm 0.01$  g precision. Dry components (laterite, OPC, WTS) were blended mechanically for 5 minutes to ensure homogeneity before gradual addition of water.

The water content was maintained within 11–15% of the dry weight of laterite to achieve optimum workability. The mixtures were compacted into steel moulds measuring 150 mm  $\times$  150 mm  $\times$  150 mm under controlled compaction pressure.

The specimens were demoulded after 24 hours and cured in water at ambient laboratory temperature ( $27 \pm 2$  °C) for 28 days prior to strength testing.



Fig. 3: Sample Brick



Fig. 4: Sample Brick

#### 2.4 Particle Size Distribution

Particle size distribution analysis was conducted using both dry and wet sieve methods in accordance with ASTM D6913. The gradation characteristics were used to determine coefficients of uniformity ( $C_u$ ) and curvature ( $C_c$ ), which provide insight into the soil's grading behaviour and suitability for block production.

#### 2.5 Atterberg Limits

The Liquid Limit (LL) and Plastic Limit (PL) were determined in accordance with BS 1377-2:1990. The Casagrande apparatus was used for LL determination, while PL was obtained by rolling soil threads to 3 mm diameter until crumbling occurred. The Plasticity Index (PI) was computed as:

$$PI = LL - PL \quad (\text{Eqn 2})$$

Shrinkage limit tests were also conducted to evaluate dimensional stability of stabilized mixtures.



## 2.6 Compressive Strength Testing

After 28 days of curing, compressive strength tests were conducted on samples Concrete Cubes with dimensions of  $300 \times 205 \times 150 \text{mm}^3$  using a Universal Testing Machine (UTM) at a loading rate of 0.5 kN/s.

The values of compressive stresses computed from the equation:

$$f_c = \frac{F}{A_c} \quad (\text{Eqn 3})$$

Measured in  $N/mm^2$ . Where  $F$  is the load value at failure measured in  $N$  and  $A_c$  the cross-sectional area perpendicular to the axis of load application measured in  $mm^2$ .



Fig. 7: Compressive Strength Testing of Sample Brick



Fig. 8: Weighing of Sample Brick in Before Compression Test

## 2.7 Optimization Procedure

Optimization was performed using the developed Scheffé's regression equations subject to practical constraints:

- Maximum compressive strength,
- Minimum plasticity index,
- Acceptable shrinkage limit range.

The optimum mixture was determined by solving the objective function within defined mixture bounds while satisfying engineering performance criteria.

## III. RESULTS AND DISCUSSION

### 3.1 Chemical Composition of Materials

The oxide compositions of the lateritic soil and Water Treatment Sludge (WTS) obtained from XRF analysis are presented in fig. 9.

The lateritic soil contained a combined silica–alumina–iron oxide ( $\text{SiO}_2 + \text{Al}_2\text{O}_3 + \text{Fe}_2\text{O}_3$ ) content of 76.22%, while WTS recorded 68.93%. These values exceed the 50% minimum requirement typically associated with pozzolanic reactivity, indicating that both materials possess significant cementitious potential when activated in the presence of calcium hydroxide from OPC hydration.

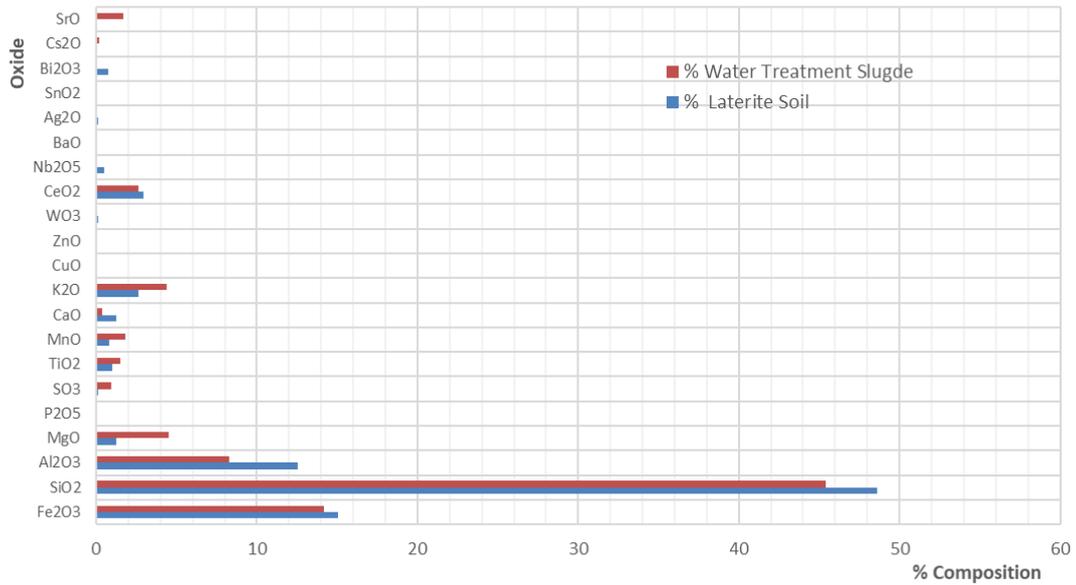


Fig 9: Mineralogical Plot of WTS and Laterite Soil

The relatively high iron oxide content explains the reddish coloration of the Abuja laterite and contributes to its binding behaviour. However, the lower combined oxide content in WTS suggests that it functions primarily as a supplementary stabilizing material rather than a primary binder. The mineralogical composition confirms that partial substitution of cement with WTS is chemically feasible without severely compromising reactivity.

### 3.2 Particle Size Distribution and Soil Classification

The particle size distribution results are presented in fig.10. The soil exhibited well-graded characteristics, with coefficients of uniformity (Cu) and curvature (Cc) falling within acceptable ranges for structural earth block production. The adequate distribution of fine and coarse fractions enhances interparticle packing, thereby improving compaction and strength performance.

The grading characteristics indicate that the untreated laterite is suitable for stabilization without excessive modification.

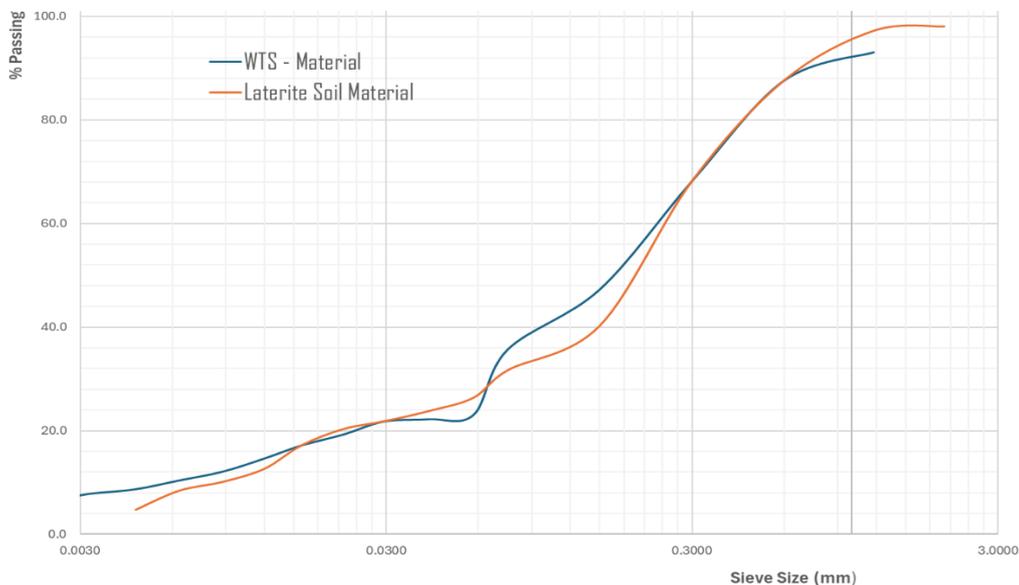


Fig. 10: Particle Size Distribution (PSD) of laterite soil and water treatment sludge (WTS)

### 3.3 Atterberg Limits of Natural Laterite

The consistency limits of the untreated laterite are presented in Table 1 and Table 2. The natural laterite exhibited moderate plasticity, with a plasticity index indicating clayey behaviour typical of tropical

residual soils. Elevated plasticity index values are associated with higher shrinkage susceptibility and cracking during drying, which may compromise structural reliability. Therefore, stabilization was necessary to optimize plasticity for block production.

Table 1: Liquid Limits Test Results of laterite soil sample

S/No.	Sample Tag	wt. of Dish (g)	wt. of Dish + wet Sample (g)	wt. of wet Sample (g)	wt. of Dish + dry Sample (g)	wt. of dry Sample (g)	Moisture Content	% Moisture Content	Penetration (mm)
1	C2	77.6	83.2	5.60	82.0	4.40	1.2	27.273	18.47
2	B1	75.2	82.3	7.10	81.0	5.80	1.3	22.414	25.88
3	E2	75.21	83.6	8.39	82.4	7.19	1.2	16.690	32.51
4	C1	76.2	85.1	8.90	84.0	7.80	1.1	14.103	35.39

Table 2: Plastic Limits Test Results of laterite soil sample

S/No.	Sample Tag	wt. of Dish (g)	wt. of Dish + wet Sample (g)	wt. of wet Sample (g)	wt. of Dish + dry Sample (g)	wt. of dry Sample (g)	Moisture Content	% Moisture Content
1	A1	80.0	82.0	2	81.6	1.6	0.4	25.00
2	E1	78.2	79.7	1.5	79.4	1.2	0.3	25.00
3	A2	80.8	82.4	1.6	82.1	1.3	0.3	23.08
4	A3	76.1	77.4	1.3	77.2	1.1	0.2	18.18

### 3.6 Compressive Strength Development

The 28-day compressive strength is a paramount engineering property that defines the load-bearing capacity and structural integrity of masonry units. The results for the stabilized lateritic bricks reveal significant variations in strength development, directly attributable to the specific blend of OPC and RHA used. The control mix, A1 (unmodified laterite), established a performance benchmark with a mean compressive strength of 3.508 MPa.

Analysis of the data indicates that the stabilization process yields a spectrum of strength outcomes, which can be categorized into three primary tiers relative to the control:

- i. High-Performance Tier (Strength Comparable to or Approaching Control): A select group of mixes successfully maintained compressive

strength near the control level. Most notably, mixes B12 and D12 achieved a mean strength of 3.469 MPa, which is 98.9% of the control's performance. Similarly, the B23, G23, and D23 group exhibited a robust strength of 3.320 MPa (94.6% of control). This demonstrates that certain OPC-RHA formulations can effectively preserve the inherent strength of the soil while conferring the additional benefits of stabilization, such as the improved moisture resistance.

- ii. Intermediate-Performance Tier (Significant Strength Reduction): A majority of the modified blends, including A2 (2.870 MPa), B13/D13 (2.782 MPa), and B14/D14 (2.356 MPa), exhibited a substantial decrease in compressive strength, ranging from 18% to

33% lower than the control. This suggests that for these specific mix designs, the Pozzolanic reaction and cementitious bonding were either sub-optimal or the RHA content may have diluted the matrix, insufficiently compensating for the reduction in densification that occurs with the addition of finer particles.

iii. Low-Performance Tier (Severe Strength Reduction): Several mixes, including A3,

B24, G24 (1.737 MPa), and A4, B34, D34 (~1.389-1.447 MPa), displayed a critical loss of strength, measuring less than 50% of the control's capacity. While some of these mixes (notably A4 and B34) were previously identified as having favorable consolidation characteristics, their exceptionally low compressive strength renders them unsuitable for structural brick production under the tested conditions.

Table 3: Compressive Strength at 28th Day of Production of Sample Bricks

S/No.	Points	Atterberg Limits Tests								Compressive Strength			
		Shrinkage Limits (%)				Liquid Limits (%)				Compressive Strength (N/mm <sup>2</sup> ) @ 28 days			
		Test 1	Test 2	Stan. Dev.	Mean %	Test 1	Test 2	Stan. Dev.	Mean %	Test 1	Test 2	Standard Dev.	Mean (MPa)
1	A1	10.228	18.959	6.1738	14.594	18.510	18.450	0.0424	18.480	2.795	3.552	0.5355	3.174
2	A2	10.160	13.219	2.1636	11.690	20.752	20.014	0.5221	20.383	1.712	2.136	0.2996	1.924
3	A3	10.127	12.455	1.6466	11.291	23.259	24.885	1.1497	24.072	2.376	1.989	0.2738	2.182
4	A4	9.233	13.510	3.0244	11.371	22.231	25.738	2.4799	23.985	2.792	2.276	0.3648	2.534
5	B12	10.133	21.595	8.1049	15.864	19.425	19.063	0.2564	19.244	2.486	3.101	0.4350	2.794
6	B13	10.170	13.205	2.1466	11.687	21.089	20.840	0.1759	20.964	1.712	2.136	0.2996	1.924
7	B14	10.362	10.114	0.1750	10.238	23.456	23.687	0.1633	23.571	1.742	1.619	0.0872	1.680
8	B23	10.424	9.796	0.4438	10.110	23.402	22.881	0.3686	23.141	1.742	1.619	0.0871	1.680
9	B24	10.127	12.455	1.6466	11.291	23.259	24.885	1.1497	24.072	2.376	1.989	0.2738	2.182
10	B34	9.666	13.066	2.4041	11.366	22.777	25.433	1.8782	24.105	2.700	2.144	0.3936	2.422
11	G12	10.133	21.595	8.1049	15.864	19.425	19.063	0.2564	19.244	2.486	3.101	0.4350	2.794
12	G13	10.170	13.205	2.1466	11.687	21.089	20.840	0.1759	20.964	1.712	2.136	0.2996	1.924
13	G14	10.362	10.114	0.1750	10.238	23.456	23.687	0.1633	23.571	1.742	1.619	0.0872	1.680
14	G23	10.424	9.796	0.4438	10.110	23.402	22.881	0.3686	23.141	1.742	1.619	0.0871	1.680
15	G24	10.127	12.455	1.6466	11.291	23.259	24.885	1.1497	24.072	2.376	1.989	0.2738	2.182
16	G34	9.666	13.066	2.4041	11.366	22.777	25.433	1.8782	24.105	2.700	2.144	0.3936	2.422
17	D34	9.666	13.066	2.4041	11.366	22.777	25.433	1.8782	24.105	2.700	2.144	0.3936	2.422
18	D23	10.424	9.796	0.4438	10.110	23.402	22.881	0.3686	23.141	1.742	1.619	0.0871	1.680
19	D24	10.424	9.796	0.4438	10.110	23.402	22.881	0.3686	23.141	1.742	1.619	0.0871	1.680
20	D12	10.133	21.595	8.1049	15.864	19.425	19.063	0.2564	19.244	2.486	3.101	0.4350	2.794

### 3.7 Statistical Model Validation

The adequacy of Scheffé's mixture model is presented in Table 4 and Table 5. The coefficient of determination ( $R^2$ ) values exceeded 0.99 for both compressive strength and Atterberg limit models, indicating excellent predictive capability. The close agreement between experimental and predicted values, confirms the robustness of the developed

polynomial equations. The high statistical significance demonstrates that mixture interaction effects were successfully captured by the fourth-degree polynomial model. The corresponding mixture models for Shrinkage limits  $X_{SL}$ , liquid limits  $X_{LL}$  and compressive strength  $X_f$  of sample earth bricks are indicated in Eqn(s). (3), (4) and (5) respectively.

$$\begin{aligned}
 X_{SL} = & 12.2094X_1 + 9.9913X_2 + 3.6432X_1X_2 - 6.5837X_1^2X_2 + 10.4551X_3 \\
 & - 4.4222X_1X_3 - 2.1770X_1^2X_3 - 4.5254X_2X_3 + 1.00445X_1X_2X_3 \\
 & + 8.5367X_2^2X_3 + 12.0373X_4 - 6.6936X_1X_4 - 0.7729X_1^2X_4 \\
 & - 9.5189X_2X_4 - 14.3630X_1X_2X_4 + 14.5641X_2^2X_4 - 0.0981X_3X_4 \\
 & + 28.0989X_1X_3X_4 - 3.7423X_2X_3X_4 + 2.40038X_3^2X_4
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 X_{LL} = & 25.8344X_1 + 21.3709X_2 + 8.2514X_1X_2 - 19.0667X_1^2X_2 + 24.3038X_3 \\
 & - 11.3864X_1X_3 - 7.5542X_1^2X_3 - 12.8140X_2X_3 + 9.1893X_1X_2X_3 \\
 & + 16.9604X_2^2X_3 + 27.3493X_4 - 16.0821X_1X_4 - 2.2087X_1^2X_4 \\
 & - 20.6607X_2X_4 - 27.4502X_1X_2X_4 + 40.8707X_2^2X_4 - 0.4929X_3X_4 \\
 & + 39.4796X_1X_3X_4 - 18.6876X_2X_3X_4 + 6.7389X_3^2X_4
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 X_f = & 3.5082X_1 + 2.8701X_2 - 1.8952X_1X_2 + 6.02483X_1^2X_2 + 1.7371X_3 - 0.4541X_1X_3 \\
 & + 2.1817X_1^2X_3 + 1.0597X_2X_3 + 4.7418X_1X_2X_3 - 1.2495X_2^2X_3 \\
 & + 1.3693X_4 - 0.4483X_1X_4 + 0.2332X_1^2X_4 + 0.9008X_2X_4 \\
 & - 3.5405X_1X_2X_4 - 4.8629X_2^2X_4 + 0.0795X_3X_4 - 0.19319X_1X_3X_4 \\
 & + 1.7660X_2X_3X_4 - 1.00754X_3^2X_4
 \end{aligned} \tag{5}$$

Table 4: Statistical Validation of  $\beta_i$  Estimate and Standard Error

S/No.	$\beta_i$ Estimate				Standard Error		
	$\beta_i$	Shrinkage Limit	Liquid Limit	Compressive Strength	Shrinkage Limit	Liquid Limit	Compressive Strength
1	$\beta_1$	12.2094	258.3440	3.5083	1.6681E-14	4.0067E-14	4.5012E-15
2	$\beta_2$	9.99131	213.7100	2.8702	1.6681E-14	4.0067E-14	4.5012E-15
3	$\beta_3$	10.4551	243.0380	1.7371	1.6681E-14	4.0067E-14	4.5012E-15
4	$\beta_4$	12.0374	273.4940	1.3694	1.6681E-14	4.0067E-14	4.5012E-15
5	$\beta_{12}$	3.6433	8.2515	-1.8952	2.5360E-12	6.0911E-12	6.8429E-13
6	$\beta_{13}$	-4.42227	-113.8650	-0.4541	2.2001E-13	5.2843E-13	5.9365E-14
7	$\beta_{14}$	-6.69361	-160.8220	-0.4483	8.9345E-13	2.1460E-12	2.4108E-13
8	$\beta_{23}$	-4.52542	-128.1410	1.0598	1.7025E-12	4.0891E-12	4.5938E-13
9	$\beta_{24}$	-9.51892	-206.6080	0.9009	1.9429E-12	4.6665E-12	5.2424E-13
10	$\beta_{34}$	-0.0981829	-0.0493	0.0795	2.3004E-13	5.5253E-13	6.2073E-14
11	$\beta_{123}$	1.00445	9.1894	4.7419	1.7087E-12	4.1040E-12	4.6105E-13
12	$\beta_{124}$	-14.363	-274.5030	-3.5405	1.9491E-12	4.6814E-12	5.2592E-13
13	$\beta_{134}$	28.099	394.7970	-0.1932	3.9011E-13	9.3698E-13	1.0526E-13
14	$\beta_{234}$	-3.74236	-186.8760	1.7660	4.0779E-13	9.7945E-13	1.1003E-13
15	$\beta_{112}$	-6.58377	-190.6670	6.0248	5.0698E-12	1.2177E-11	1.3680E-12
16	$\beta_{113}$	-2.17701	-7.5543	2.1818	4.1933E-13	1.0072E-12	1.1315E-13
17	$\beta_{114}$	-0.772977	-2.2088	0.2332	1.7821E-12	4.2805E-12	4.8088E-13

18	$\beta_{223}$	8.53678	169.6050	-1.2495	3.4050E-12	8.1783E-12	9.1877E-13
19	$\beta_{224}$	14.5642	408.7080	-4.8630	3.8857E-12	9.3329E-12	1.0485E-12
20	$\beta_{334}$	2.40039	6.7390	-1.0075	4.6774E-13	1.1234E-12	1.2621E-13

Table 5: Statistical Validation of t-Statistic and P-Value

S/No.	$\beta_i$	t-Statistic			P-Value		
		Shrinkage Limit	Liquid Limit	Compressive Strength	Shrinkage Limit	Liquid Limit	Compressive Strength
1	$\beta_1$	7.3192E+14	6.4479E+14	7.7941E+14	1.2436E-229	9.4494E-229	4.5478E-230
2	$\beta_2$	5.9895E+14	5.3339E+14	6.3765E+14	3.0749E-228	1.9651E-227	1.1292E-228
3	$\beta_3$	6.2675E+14	6.0659E+14	3.8593E+14	1.4877E-228	2.5106E-228	3.4835E-225
4	$\beta_4$	7.2160E+14	6.8260E+14	3.0422E+14	1.5606E-229	3.7968E-229	1.5666E-223
5	$\beta_{12}$	1.4366E+12	1.3547E+12	-2.7696E+12	2.5615E-186	6.5567E-186	7.0366E-191
6	$\beta_{13}$	-2.0101E+13	-2.1548E+13	-7.6501E+12	1.1878E-204	3.9051E-205	6.1292E-198
7	$\beta_{14}$	-7.4918E+12	-7.4942E+12	-1.8597E+12	8.5633E-198	8.5206E-198	4.1209E-188
8	$\beta_{23}$	-2.6581E+12	-3.1337E+12	2.3070E+12	1.3577E-190	9.7528E-192	1.3104E-189
9	$\beta_{24}$	-4.8995E+12	-4.4275E+12	1.7184E+12	7.6501E-195	3.8681E-194	1.4585E-187
10	$\beta_{34}$	-4.2680E+11	-8.9219E+11	1.2814E+12	6.9574E-178	5.2325E-183	1.5971E-185
11	$\beta_{123}$	5.8786E+11	2.2391E+12	1.0285E+13	4.1468E-180	2.1125E-189	5.3816E-200
12	$\beta_{124}$	-7.3692E+12	-5.8637E+12	-6.7321E+12	1.1153E-197	4.3186E-196	4.7392E-197
13	$\beta_{134}$	7.2029E+13	4.2135E+13	-1.8354E+12	1.6067E-213	8.5466E-210	5.0874E-188
14	$\beta_{234}$	-9.1772E+12	-1.9080E+13	1.6050E+13	3.3317E-199	2.7347E-204	4.3504E-203
15	$\beta_{112}$	-1.2986E+12	-1.5658E+12	4.4041E+12	1.2893E-185	6.4612E-187	4.2105E-194

16	$\beta_{113}$	- 5.1917E+12	-7.5005E+12	1.9283E+13	3.0279E- 195	8.4063E- 198	2.3087E-204
17	$\beta_{114}$	- 4.3374E+11	-5.1602E+11	4.8497E+11	5.3759E- 178	3.3376E- 179	9.0092E-179
18	$\beta_{223}$	2.5072E+12	2.0739E+12	-1.3600E+12	3.4606E- 190	7.2050E- 189	6.1593E-186
19	$\beta_{224}$	3.7482E+12	4.3792E+12	-4.6381E+12	5.5585E- 193	4.6101E- 194	1.8391E-194
20	$\beta_{334}$	5.1319E+12	5.9985E+12	-7.9830E+12	3.6442E- 195	3.0020E- 196	3.1004E-198

### 3.8 Mixture Optimization

After conducting laboratory tests on the thirty-five (35) mixture formulations to determine the Atterberg limits and compressive strength of the stabilized earth brick specimens, in line with Scheffé's optimization model, the optimum mixture proportions were identified based on the respective shrinkage limit, liquid limit, and compressive strength responses. The analytical results indicate optimal pseudo-component

proportions of [0.2322, 0.4386, 0.0062, 0.3229], [0.4041, 0.3031, 0.2904], and [0.5285, 0.38910, 0.0822, 0.00011] corresponding to the shrinkage limit, liquid limit, and compressive strength, respectively. These values define the ideal ratios of water, Ordinary Portland Cement (OPC), Water Treatment Sludge (WTS), and lateritic soil within the mix designs. Tables 6, 7, and 8 present the optimal mixture configurations for each respective response.

Table 6: Optimal mixture configurations for corresponding Shrinkage Limit responses

S/No	Component	Optimal Mixture Shrinkage Limit								
		$X_i$	Mix-1	Mix-2	Mix-3	Mix-4	Mix-5	Mix-6	Mix-7	Mix-8
1	Water	0.23222	0.323	0.314	0.335	0.333	0.323	0.318	0.322	0.322
2	Ordinary Portland Cement	0.43861	0.173	0.164	0.185	0.183	0.173	0.168	0.172	0.172
3	Water Treatment Sludge	0.00621	0.471	0.427	0.527	0.519	0.468	0.452	0.468	0.468
4	Laterite Soil	0.32296	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	Shrinkage Limit (%)	1.00000	10.2354	10.1874	10.6183	10.6423	10.1058	10.2783	10.2815	10.2815

Table 7: Optimal mixture configurations for corresponding Liquid Limit responses

S/No.	Component	Optimal Mixture Liquid Limit								
		$X_i$	Mix-1	Mix-2	Mix-3	Mix-4	Mix-5	Mix-6	Mix-7	Mix-8
1	Water	0.40410	0.317	0.311	0.334	0.332	0.328	0.316	0.321	0.321
2	Ordinary Portland Cement	0.30309	0.167	0.161	0.184	0.182	0.178	0.166	0.171	0.171

3	Water Treatment Sludge	0.00238	0.445	0.414	0.523	0.517	0.493	0.444	0.462	0.462
4	Laterite Soil	0.29043	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	Liquid Limit (%)	1.00000	21.568	21.4566	23.328	23.1423	22.2309	22.0156	21.5804	21.5804

Table 8: Optimal mixture configurations for corresponding Compressive Strength responses

S/No.	Component	Optimal Mixture Compressive Strength								
		X <sub>i</sub>	Mix-1	Mix-2	Mix-3	Mix-4	Mix-5	Mix-6	Mix-7	Mix-8
1	Water	0.528536	0.301	0.307	0.339	0.326	0.338	0.317	0.323	0.323
2	Ordinary Portland Cement	0.389107	0.151	0.157	0.189	0.176	0.188	0.167	0.173	0.173
3	Water Treatment Sludge	0.082243	0.370	0.397	0.547	0.487	0.541	0.447	0.471	0.471
4	Laterite Soil	0.000114	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	Comp. Strgth (MPa)	1.000000	3.3276	3.15163	1.85762	2.41824	1.84216	2.658	2.55146	2.55146

#### IV. CONCLUSIONS

Based on the experimental investigation and Scheffé mixture optimization of cement–WTS stabilized Abuja laterite, the following conclusions are drawn:

1. Abuja laterite demonstrated strong stabilization potential, with a combined SiO<sub>2</sub> + Al<sub>2</sub>O<sub>3</sub> + Fe<sub>2</sub>O<sub>3</sub> content of 76.22%, indicating favourable mineralogical characteristics for cementitious reactions.
2. Water Treatment Sludge (WTS) exhibited moderate pozzolanic suitability, with 68.93% combined oxide content, supporting its role as a supplementary stabilizing material.
3. Cement–WTS stabilization significantly improved consistency characteristics, reducing plasticity index and shrinkage behaviour. The reduction in plasticity is attributed to flocculation–agglomeration mechanisms and cementitious bond formation.

4. The control mixture achieved a 28-day compressive strength of 3.508 MPa (Table 4.10), while optimized WTS-modified mixtures attained up to 98.9% of this value, confirming that partial cement replacement did not substantially compromise structural performance.
5. Scheffé's fourth-degree mixture model demonstrated excellent predictive capability, with R<sup>2</sup> values exceeding 0.99, validating the robustness of the statistical optimization approach.
6. The optimal formulation, containing approximately 39% OPC and 8% WTS within the mixture system, provided a balanced improvement in strength, plasticity, and shrinkage resistance, while reducing cement demand.

Overall, controlled incorporation of WTS (≤ 8%) can partially substitute cement in stabilized earth blocks without significant loss in structural reliability, thereby promoting sustainable and cost-effective construction practices.

## V. RECOMMENDATION

Based on the findings of this study, the following recommendations are proposed:

1. WTS replacement should not exceed approximately 8% within the investigated mixture bounds to avoid strength dilution effects.
2. Field-scale validation studies should be conducted to assess long-term durability performance under environmental exposure conditions typical of tropical climates.
3. Microstructural analysis (e.g., SEM or XRD) is recommended in future work to further elucidate hydration and pozzolanic reaction mechanisms.
4. Regulatory agencies and construction stakeholders in Nigeria should consider incorporating WTS-based stabilization guidelines into sustainable building material standards.

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## REFERENCES

- [1] ASTM International. (2017). *ASTM C67: Standard test methods for sampling and testing brick and structural clay tile*. ASTM International.
- [2] British Standards Institution. (2011). *BS EN 771-1: Specification for masonry units*. BSI.
- [3] Das, B. M. (2013). *Advanced soil mechanics* (4th ed.). CRC Press.
- [4] Das, B. M., & Sobhan, K. (2014). *Principles of geotechnical engineering* (8th ed.). Cengage Learning.
- [5] Frías, M., Villar-Cociña, E., & Savastano, H., Jr. (2011). Characterization and properties of blended cement matrices containing water treatment sludge. *Cement and Concrete Composites*, 33(7), 709–715.
- [6] Ganesan, K., Rajagopal, K., & Thangavel, K. (2008). Rice husk ash blended cement: Assessment of optimal level of replacement for strength and permeability properties of concrete. *Cement and Concrete Composites*, 30(7), 561–569.
- [7] Lambe, T. W., & Whitman, R. V. (1969). *Soil mechanics*. Wiley.
- [8] Monzó, J., Payá, J., & Borrachero, M. V. (2003). Utilization of sewage sludge ash in cement-based materials. *Waste Management*, 23(4), 373–381.
- [9] Neville, A. M. (2011). *Properties of concrete* (5th ed.). Pearson Education.
- [10] Olivier, J. G. J., Janssens-Maenhout, G., Muntean, M., & Peters, J. A. H. W. (2016). *Trends in global CO<sub>2</sub> emissions*. PBL Netherlands Environmental Assessment Agency.
- [11] Scheffé, H. (1958). Experiments with mixtures. *Journal of the Royal Statistical Society: Series B (Methodological)*, 20(2), 344–360.
- [12] Terzaghi, K., Peck, R. B., & Mesri, G. (1996). *Soil mechanics in engineering practice* (3rd ed.). Wiley.
- [13] United Nations Environment Programme. (2019). *Sustainable resource management and circular economy*. UNEP.
- [14] Vouk, D., Nakic, D., & Stirmer, N. (2014). Use of sludge generated at water treatment plants in construction materials. *Materials and Structures*, 47(4), 593–602.
- [15] Zhang, M. H., & Malhotra, V. M. (1996). High-performance concrete incorporating rice husk ash as a supplementary cementing material. *Cement and Concrete Research*, 26(10), 1433–1443.