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Conditional Value-at-Risk-Based Portfolio Optimization for Multi-Asset Treasury Management in Nigerian Banks

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Abstract- Portfolio optimization remains a central problem in financial mathematics, with the classical Mean–Variance framework widely applied in asset allocation decisions. De- spite its popularity, variance as a risk measure fails to adequately capture extreme losses and tail risk, which are critical in volatile financial markets. This study extends the traditional portfolio optimization framework by incorporating Conditional Value at Risk (CVaR) as an alternative risk measure and explicitly accounting for transaction costs. Using historical asset return data of the Nigerian financial market, portfolios are constructed under both Mean–Variance and Mean–CVaR optimization frameworks, evaluated under frictionless conditions and scenarios involving proportional trans- action costs. The optimization problems are solved numerically using Python-based computational techniques, and efficient frontiers are generated for comparative analysis. Furthermore, stress testing is conducted under adverse market scenarios, including foreign exchange shocks and interest rate spikes, to assess portfolio robustness. The results indicate that Mean–CVaR optimized portfolios exhibit greater stability and lower downside risk under stressed conditions, particularly when transaction costs are present. These findings highlight the practical relevance of CVaR-based optimization in realistic portfolio management and risk control settings.

Index Terms— Portfolio Optimization, Conditional Value at Risk, Mean–Variance Model, Transaction Costs

I. INTRODUCTION

Portfolio optimization is a fundamental problem in financial mathematics and investment theory, concerned with the optimal allocation of wealth among a set of financial assets in order to balance expected return against risk. Since the seminal work of Markowitz, the Mean–Variance framework has remained the cornerstone of modern portfolio theory due to its analytical tractability and intuitive appeal. In this frame- work, risk is quantified by the variance of portfolio returns, and optimal portfolios are obtained by minimizing variance for a given level of expected return. Despite its widespread adoption, the Mean–Variance model exhibits notable limitations, particularly in its inability to adequately capture down- side risk and extreme market losses.

Financial return distributions are often characterized by skew- ness, heavy tails, and volatility clustering, features that violate the normality assumptions underlying variance-based risk measures. As a result, variance penalizes both upward and limitation has motivated the development of alternative risk measures that focus explicitly on tail risk. Among these, Conditional Value at Risk (CVaR) has gained significant attention due to its coherence properties and its ability to measure expected losses beyond a specified confidence level.

In addition to the choice of risk measure, realistic portfolio optimization must account for market frictions such as trans- action costs. Ignoring transaction costs can lead to excessive portfolio turnover and overstated performance, thereby reducing the practical applicability of theoretical optimization results. Transaction costs introduce nonlinearity into the optimization problem and can substantially alter the structure of optimal portfolios. Consequently, incorporating transaction costs is essential for bridging the gap between theoretical port- folio models and real-world investment practice.

This study investigates portfolio optimization under both the traditional Mean–Variance framework and the Mean–CVaR framework, with explicit consideration of proportional trans- action costs. Using historical asset return data, optimal port- folios are constructed under frictionless conditions and un- der scenarios that include transaction costs. The optimization problems are solved numerically using Python-based computational methods, and efficient frontiers are generated to facilitate comparative analysis between the competing frame- works.

Furthermore, the robustness of the optimized portfolios is examined through stress testing under adverse market conditions, including foreign exchange shocks and interest rate spikes. By comparing portfolio losses under stressed scenarios, the study provides insights into the relative stability and risk resilience of Mean–Variance and Mean–CVaR optimized portfolios. The findings contribute to the growing literature on risk-sensitive portfolio optimization and offer practical implications for investors and risk managers operating in environments characterized by market volatility and transaction costs

II. IDENTIFY, RESEARCH AND COLLECT IDEA

The theory of portfolio optimization originates from the seminal work of Markowitz (1952), who introduced the Mean–Variance framework as a quantitative approach to asset allocation under uncertainty. The model establishes that rational investors seek to maximize expected return while minimizing portfolio variance, leading to the concept of the efficient frontier. Although the Mean–Variance paradigm re-downward deviations from the mean equally, despite investors 1 mains foundational in modern finance, its reliance on variance typically being more concerned with adverse outcomes. This as a symmetric measure of risk limits its effectiveness in environments characterized by non-normal return distributions and extreme market events.

Subsequent developments in financial risk management emphasized the need for downside-focused risk measures. Value at Risk (VaR) emerged as a widely adopted metric for quantifying potential losses at a given confidence level and time horizon. Its intuitive interpretation and regulatory acceptance contributed to its widespread use in banking and portfolio management. However, VaR suffers from significant theoretical limitations, particularly its lack of sub-additivity in certain distributions, which undermines the diversification principle and renders it inconsistent with the axioms of coherent risk measures.

To address these limitations, the concept of coherent risk measures was formalized by Artzner (1999), establishing key properties such as monotonicity, sub-additivity, positive homogeneity, and translation invariance. Conditional Value at Risk (CVaR), also referred to as Expected Shortfall, was subsequently developed as a coherent alternative to VaR that captures the expected loss in the tail of the loss distribution be- yond the VaR threshold. Unlike variance and VaR, CVaR provides a more comprehensive assessment of extreme downside risk and is particularly suitable for portfolio optimization in volatile and fat-tailed financial markets.

Rockafellar and Uryasev advanced the practical implementation of CVaR by reformulating the CVaR minimization problem into a convex optimization framework solvable via linear programming. This transformation significantly improved computational efficiency and enabled the application of CVaRpirical studies that jointly examine tail-risk optimization and transaction cost effects within a computational framework suitable for practical implementation. This study contributes to the existing literature by providing a comparative analysis of Mean–Variance and Mean–CVaR portfolio optimization under both frictionless and transaction cost scenarios, sup- ported by numerical optimization and stress testing. By doing so, it bridges the gap between theoretical risk modeling and realistic portfolio decision-making in financial mathematics

III. METHODOLOGY

This study adopts a quantitative financial mathematics frame- work to construct and evaluate optimal portfolios under both Mean–Variance and Mean–CVaR optimization paradigms. The methodological structure integrates risk modelling, trans- action cost considerations, and computational optimization to generate implementable portfolio allocation strategies under realistic market conditions.

A. Data Description and Return Series Construction

The empirical analysis is based on Nigerian financial market historical asset return series obtained over a specified sample period where Nigerian banks serve as the investors. Let P(i) t denote the price of asset i at time t. The logarithmic return of asset i is computed as:

$$r_t^{(i)} = \ln\left(\frac{P_t^{(i)}}{P_{t-1}^{(i)}}\right), \qquad (1)$$

where $r_t^{(i)}$ represents the continuously compounded return. The return matrix $R \in \mathbb{R}^{T\times N}$ is constructed, where T denotes the number of observations and N represents the number of assets. The expected return vector μ and covariance matrix Σ are estimated using sample statistics.

B. Mean–Variance Portfolio Optimization Model

The classical Mean–Variance framework is employed as a benchmark model. Let $w = (w1, w2,\ldots, wN)^\top$ denote the portfolio weight vector such that $\sum_{i=1}^{N} wi = 1$ and $wi \geq 0$. The portfolio expected return is defined as:

$$E(R_p) = w^\top \mu, \qquad (2)$$

while the portfolio variance, which serves as the traditional risk measure, is expressed as:

$$\sigma_p^2 = w^\top \Sigma w. \qquad (3)$$

The Mean–Variance optimization problem is therefore formulated as:

$$\min_{w} \quad w^\top \Sigma w \qquad (4)$$

$$\text{subject to} \quad w^\top \mu \geq \mu_0, \quad \mathbf{1}^\top w = 1, \quad w \geq 0, \qquad (5)$$

where μ0 represents the target expected return and 1 is a vector of ones. This formulation generates the efficient frontier under the assumption of normally distributed returns.

C. Conditional Value at Risk (CVaR) Optimization Framework

To overcome the limitations of variance in capturing extreme losses, Conditional Value at Risk (CVaR) is adopted as the primary downside risk measure. For a portfolio loss random variable L and confidence level $\alpha \in (0,1)$, CVaR is defined as the expected loss conditional on losses exceeding the Value at Risk (VaR):

$$\mathrm{CVaR}_\alpha(L) = \mathbb{E}\left[L \mid L \geq \mathrm{VaR}_\alpha(L)\right]. \qquad (6)$$

Following the Rockafellar–Uryasev approach, the CVaR minimization problem is reformulated into a convex optimization model using an auxiliary variable η and slack variables ξi. For N return scenarios, the optimization problem is given by:

$$\min_{w,\eta,\xi} \quad \eta + \frac{1}{(1-\alpha)N}\sum_{i=1}^{N} \xi_i \qquad (7)$$

$$\text{subject to} \quad \xi_i \geq 0, \quad \xi_i \geq -w^\top r_i - \eta, \quad i = 1,2,\ldots,N, \qquad (8)$$

$$\mathbf{1}^\top w = 1, \quad w \geq 0, \qquad (9)$$

where r_i denotes the asset return vector under scenario i, η represents the VaR level, and ξi captures losses exceeding the VaR threshold. This linear programming formulation enables efficient computation and direct minimization of tail risk in portfolio selection.

D. Incorporation of Transaction Costs

To ensure practical applicability, proportional transaction costs are incorporated into the optimization framework. Let c denote the transaction cost rate and Δwi represent the change in portfolio weight of asset i. The total transaction cost is modelled as:

$$TC = c\sum_{i=1}^{N} |\Delta w_i|. \qquad (10)$$

The inclusion of transaction costs adjusts the effective portfolio return and reduces excessive rebalancing, thereby producing more stable and implementable portfolios in real financial markets.

E. Computational Implementation

The optimization models are implemented using Python-based numerical optimization techniques. Both Mean–Variance and Mean–CVaR models are solved under frictionless and transaction cost scenarios. Efficient frontiers are constructed to evaluate the risk–return trade-offs, while stress testing

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under adverse market conditions such as exchange rate shocks and interest rate spikes is conducted to assess portfolio robustness and downside risk behaviour.

IV. MODEL FORMULATION AND OPTIMIZATION FRAMEWORK

This section presents the mathematical formulation of the portfolio optimization models employed in this study. The framework integrates the classical Mean–Variance model, the Mean–CVaR optimization approach, and the inclusion of transaction costs to ensure realistic and implementable portfolio allocation under market frictions.

A. Portfolio Return and Loss Representation

Consider a portfolio consisting of $\boldsymbol{N}$assets with weight vector
$\boldsymbol{w} = (\boldsymbol{w_1}, \boldsymbol{w_2}, \dots, \boldsymbol{w_N})^\top$such that $\sum_{i=1}^{N} \boldsymbol{w_i} = \boldsymbol{1}$and $\boldsymbol{w_i} \geq \boldsymbol{0}$.

Let $\boldsymbol{r_t} = \left(\boldsymbol{r_t^{(1)}}, \boldsymbol{r_t^{(2)}}, \dots, \boldsymbol{r_t^{(N)}}\right)^\top$denote the vector of asset returns at time $\boldsymbol{t}$. The portfolio return at time $\boldsymbol{t}$is given by:

$$\boldsymbol{R_{p,t}} = \boldsymbol{w}^\top \boldsymbol{r_t} \qquad (11)$$

The corresponding portfolio loss is defined as:

$$\boldsymbol{L_t} = -\boldsymbol{w}^\top \boldsymbol{r_t} \qquad (12)$$

where positive values of $\boldsymbol{L_t}$indicate portfolio losses. This loss representation forms the basis for tail risk measurement using Value at Risk (VaR) and Conditional Value at Risk (CVaR).

B. Mean–Variance Optimization Formulation

The Mean–Variance model serves as the benchmark optimization framework. The objective is to minimize portfolio risk measured by variance while achieving a minimum target return. The optimization problem is expressed as:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w} \qquad (13)$$

subject to

$$\boldsymbol{w}^\top \boldsymbol{\mu} \geq \boldsymbol{\mu_0}, \boldsymbol{1}^\top \boldsymbol{w} = \boldsymbol{1}, \boldsymbol{w} \geq \boldsymbol{0}, \qquad (14)$$

where $\boldsymbol{\mu}$represents the expected return vector, $\boldsymbol{\Sigma}$denotes the covariance matrix of asset returns, and $\boldsymbol{\mu_0}$is the minimum acceptable return level. This formulation generates the efficient frontier under variance-based risk assessment.

C. Mean–CVaR Optimization Model

To better capture extreme downside risk, the study adopts a Mean–CVaR optimization framework. For a confidence level $\boldsymbol{\alpha} \in (\boldsymbol{0}, \boldsymbol{1})$, CVaR measures the expected loss conditional on losses exceeding the Value at Risk threshold. Using a scenario-based approach with $\boldsymbol{N}$return observations, the CVaR minimization problem is formulated following the Rockafellar–Uryasev linear programming representation:

$$\min_{w,\eta,\xi} \boldsymbol{\eta} + \frac{\boldsymbol{1}}{(\boldsymbol{1} - \boldsymbol{\alpha})\boldsymbol{N}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{\xi_i} \qquad (15)$$

subject to

$$\boldsymbol{\xi_i} \geq \boldsymbol{0}, \boldsymbol{\xi_i} \geq -\boldsymbol{w}^\top \boldsymbol{r_i} - \boldsymbol{\eta}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{N}, \qquad (16)$$

$$\boldsymbol{1}^\top \boldsymbol{w} = \boldsymbol{1}, \boldsymbol{w} \geq \boldsymbol{0}, \qquad (17)$$

where $\boldsymbol{\eta}$represents the Value at Risk (VaR) at confidence level $\boldsymbol{\alpha}$, and $\boldsymbol{\xi_i}$are auxiliary slack variables capturing tail losses beyond the VaR threshold. This convex optimization structure enables efficient computation and direct minimization of tail risk.

D. Mean–CVaR Optimization with Return Constraint

To ensure that the optimized portfolio maintains a desirable level of expected return, a return constraint is incorporated into the CVaR framework. The constrained optimization problem is expressed as:

$$\min_{w,\eta,\xi} \boldsymbol{\eta} + \frac{\boldsymbol{1}}{(\boldsymbol{1} - \boldsymbol{\alpha})\boldsymbol{N}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{\xi_i} \qquad (18)$$

subject to

$$\boldsymbol{w}^\top \boldsymbol{\mu} \geq \boldsymbol{\mu_0}, \boldsymbol{1}^\top \boldsymbol{w} = \boldsymbol{1}, \boldsymbol{w} \geq \boldsymbol{0}, \qquad (19)$$

$$\boldsymbol{\xi_i} \geq \boldsymbol{0}, \boldsymbol{\xi_i} \geq -\boldsymbol{w}^\top \boldsymbol{r_i} - \boldsymbol{\eta}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{N}. \qquad (20)$$

This formulation simultaneously controls expected return and tail risk, making it more suitable for risk-sensitive portfolio management compared to variance-based models.

E. Transaction Cost Integration in Portfolio Optimization

In practical portfolio management, transaction costs significantly affect rebalancing decisions and portfolio performance. Let $\boldsymbol{c}$denote the proportional transaction cost rate and w^{old} represent the previous portfolio weights. The transaction cost function is defined as:

$$TC = c\sum_{i=1}^{N} | w_i - w_i^{old} | \qquad (21)$$

The net portfolio return after accounting for transaction costs is therefore adjusted as:

$$R_p^{net} = w^{\top}\mu - TC. \qquad (22)$$

The inclusion of transaction costs discourages excessive turnover and leads to more stable portfolio allocations that are implementable in real financial markets.

F. Stress Testing and Scenario-Based Risk Evaluation
To evaluate the robustness of the optimized portfolios, stress testing is conducted under adverse market scenarios such as exchange rate shocks and interest rate spikes. Let r_t^{shock} denote the stressed return vector. The stressed portfolio loss is computed as:

$$L_t^{shock} = -w^{\top} r_t^{shock}. \qquad (23)$$

Comparative analysis of losses under normal and stressed conditions enables assessment of portfolio resilience and the effectiveness of CVaR optimization in mitigating extreme downside risk. This scenario-based evaluation framework provides deeper insight into the stability and practical performance of optimized portfolios under volatile financial conditions.

V. RESULTS AND DISCUSSION

This section presents empirical results from the portfolio optimization frameworks considered in this study. The analysis compares the traditional Mean–Variance model against Mean–CVaR models under frictionless conditions and under proportional transaction costs. In addition, stress testing is used to evaluate robustness under adverse market scenarios relevant to Nigerian financial conditions.

A. Nigerian Bank Treasury Dataset and Asset Universe
The empirical study uses historical data spanning January 2020 to December 2024, comprising 1,305 trading-day observations. The asset universe reflects typical Nigerian bank treasury exposures and is constructed to include government fixed-income securities, bank equities, and foreign exchange-linked instruments. All price series are converted into consistent return measures using log-returns and aligned on a common sample period before optimization.

Table 1: Nigerian Bank Treasury Asset Universe (Illustrative Grouping)

Asset Class	Examples (as used in study)	Treasury Rationale
Government Securities	Treasury Bills, FGN Bonds	Liquidity management, capital preservation, regulatory compliance
Bank Equities (NGX)	UBA, GTB and other bank stocks	Return enhancement, equity risk exposure, diversification
Foreign Exchange-linked	FX instruments / proxies	FX risk management, macro hedge, currency exposure

The dataset design reflects the multi-asset structure of Nigerian bank treasury portfolios where liquidity requirements and macroeconomic volatility (exchange rate and interest rate movements) strongly influence allocation decisions.

B. Efficient Frontier Comparison Efficient frontiers are constructed for Mean–Variance and Mean–CVaRmodelstoevaluatetherisk–return trade-off across optimization paradigms. The Mean–Variance frontier captures volatility-based risk, while the Mean–CVaR frontier captures tail-loss sensitivity. In volatile markets characterized by fat tails, CVaR frontiers provide more relevant risk control because they directly penalize extreme downside realizations rather than overall dispersion.

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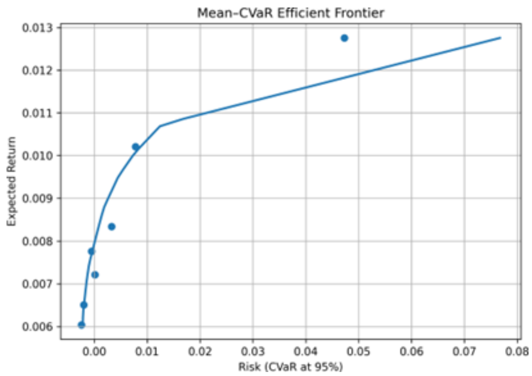

Figure 1: Mean–CVaR Efficient Frontier (Frictionless Case)

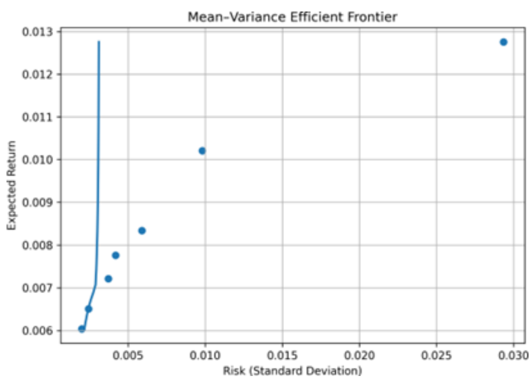

Figure 2: Mean–Variance Efficient Frontier (Frictionless Case)

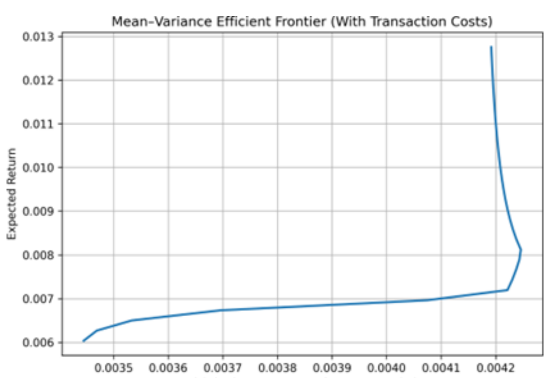

Figure 3: Mean–Variance Efficient Frontier (With Transaction Costs)

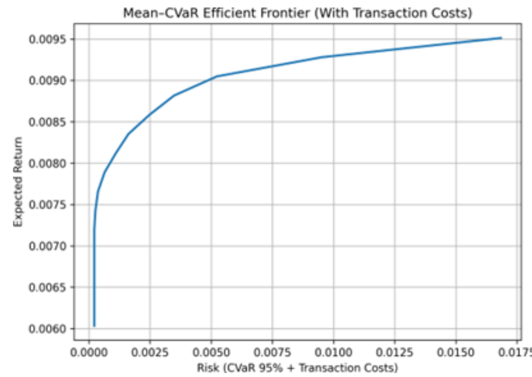

Figure 4: Mean–CVaR Efficient Frontier (With Transaction Costs)

Transaction costs shift the frontier downward by reducing net returns and discouraging aggressive rebalancing, which improves implementability. In particular, the transaction-cost-aware frontier is associated with reduced turnover and more stable weight trajectories compared to the frictionless counterpart.

C. Portfolio Performance Summary

Portfolio performance is summarized using annualized return, volatility, and Sharpe ratio. Results show that CVaR-based optimization with transaction costs produces a favourable balance between return generation and tail-risk control. In the study, the CVaR-optimized portfolio with 0.1% transaction costs attains an annual return of 17.31% with volatility of 4.10% and Sharpe ratio of 1.17, indicating strong risk-adjusted performance alongside improved practicality through turnover reduction.

Table 2: Summary Performance Metrics (Annualized)

Model	Return (%)	Volatility (%)	Sharpe Ratio
Mean–Variance (Benchmark)	17.95	4.42	1.24
Mean–CVaR (Frictionless)	16.95	4.06	1.10
Mean–CVaR (With 0.1% TC)	17.31	4.10	1.17

Table 2 highlights that CVaR optimization remains competitive on risk-adjusted performance while providing better control of extreme downside outcomes, particularly when realistic trading frictions are considered.

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D. Optimal Allocation and Economic Interpretation

The optimized allocation is dominated by government securities, reflecting the central role of liquidity, capital preservation, and regulatory constraints in Nigerian bank treasury management. In the study, the optimal CVaR portfolio with transaction costs allocates approximately 94.04% to government securities, 5.32% to bank equities, and 0.63% to foreign exchange exposures. This allocation structure supports downside protection but also indicates concentration risk that may arise under strong liquidity/regulatory priorities.

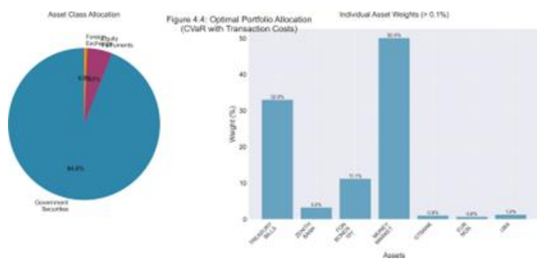

Figure 5: Optimal Portfolio Weights under Mean–CVaR with Transaction Costs

Within the equity segment, the allocation favours relatively stronger-performing banking equities (e.g., UBA receiving a notable share within the equity bucket), consistent with the role of selective equity exposure as a return-enhancement overlay rather than a dominant treasury holding.

E. Tail Risk and Risk Measure Comparison

Risk evaluation incorporates multiple measures including VaR, CVaR, maximum loss, and average loss. The empirical distribution of returns exhibits fat-tail behaviour, implying that extreme losses occur more frequently than Gaussian models predict. Under such conditions, CVaR provides a more informative and conservative measure of risk because it quantifies expected tail losses beyond the VaR threshold.

Table 3: Tail Risk Indicators (Illustrative Reporting)

Risk Measure	Estimate (%)	Interpretation
VaR (95%)	-0.35	Loss threshold at confidence level
CVaR (95%)	-0.48	Average loss beyond VaR (tail expectation)
Maximum Loss	-0.89	Worst realized loss in sample / scenario
Average Loss	-0.18	Mean of negative returns (downside average)

These measures collectively demonstrate that CVaR-based optimization offers stronger protection against tail events, which is essential in markets exposed to exchange-rate instability, interest-rate shocks, and episodic liquidity disruptions.

F. Stress Testing: FX and Interest Rate Shock Scenarios

Stress testing is implemented to evaluate portfolio resilience under adverse economic conditions. Scenarios include an exchange-rate shock and an interest-rate spike, which are relevant for Nigerian macro-financial dynamics. Stress losses are computed as portfolio losses under shocked return vectors and compared across optimization models.

Table 4: Stress Test Portfolio Losses (CVaR Portfolio with Transaction Costs)

Scenario	Portfolio Loss (%)
Baseline (current conditions)	0.00
Oil Price Shock (-40%)	1.19
Currency Crisis (25% devaluation)	-0.27
Interest Rate Shock (+400bps)	-3.79
Combined Shock	-2.87

The stress results (Table 4) indicate that the CVaR-optimized portfolio exhibits strong resilience, with maximum loss of -3.79% under the combined shock scenario. Negative loss (gain) under currency devaluation reflects the portfolio's USD-denominated assets, highlighting potential hedging benefits.

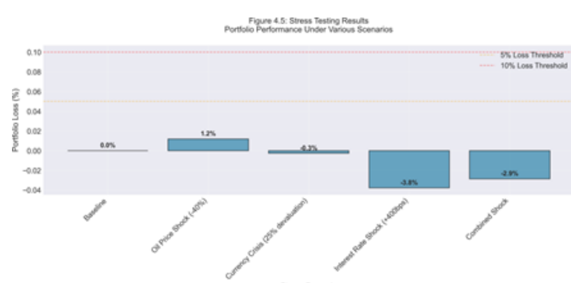

Figure 6: Stress test results showing portfolio losses under transaction costs

G. Practical Implications for Nigerian Bank Treasury Management

The results provide practical implications for treasury managers. First, CVaR-based frameworks enhance protection against extreme losses and offer better alignment with downside-risk objectives. Second, transaction cost integration reduces turnover and improves implementability. Finally, the dominance of government securities underscores a liquidity–diversification trade-off: stronger liquidity and regulatory alignment may increase concentration risk, motivating careful monitoring and policy refinement where appropriate.

VI. CONCLUSION

This study developed and implemented a quantitative portfolio optimization framework based on Conditional Value at Risk (CVaR) within a financial mathematics context, with explicit consideration of transaction costs and realistic market constraints. The comparative analysis between the traditional Mean–Variance model and the Mean–CVaR framework demonstrates that variance-based optimization, while analytically convenient, is insufficient for capturing extreme downside risks prevalent in volatile financial markets.

Empirical findings indicate that the CVaR-optimized portfolio provides superior tail-risk protection and improved stability under stressed market conditions. The integration of proportional transaction costs further enhances the practical relevance of the optimization results by reducing excessive portfolio turnover and producing more implementable allocation strategies. In particular, the optimized portfolios exhibit a strong preference for government securities, reflecting the liquidity requirements and regulatory considerations that characterize Nigerian bank treasury management.

The stress testing analysis under adverse scenarios such as exchange rate shocks and interest rate spikes confirms the robustness of the Mean–CVaR framework in mitigating extreme losses. Compared to the Mean–Variance model, the CVaR-based approach offers a more conservative and risk-sensitive allocation, especially in markets exhibiting fat-tailed return distributions and macroeconomic instability.

From a practical perspective, the results suggest that Nigerian bank treasury departments can significantly enhance risk management practices by adopting CVaR-based optimization techniques that explicitly account for tail risk, transaction costs, and regulatory constraints. Such an approach aligns with modern financial risk management principles and provides a more resilient framework for multi-asset treasury portfolio construction.

Future research may extend this framework to dynamic multi-period optimization, incorporate liquidity-adjusted risk measures, and integrate machine learning techniques for scenario generation and return forecasting. Additionally, the inclusion of broader risk dimensions such as credit risk and liquidity risk would further strengthen the applicability of the model to comprehensive bank treasury management systems.

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