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A New Exponential Ratio-Type Estimator for Population Mean Using Two Auxiliary Variables in Double Sampling

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Abstract- In this study, a new exponential ratio-type estimator for estimating the population mean using two auxiliary variables in double sampling is proposed. The bias and mean squared error (MSE) of the proposed estimator are derived. The efficiency of the proposed estimator is compared theoretically with some existing estimators such as the usual sample mean estimator, the classical ratio estimator, and other exponential estimators. An empirical study using real-life data is conducted to examine the performance of the estimator. The results indicate that the proposed estimator has the smallest mean square error and the highest relative efficiency among the estimators considered. Hence, the estimator is recommended for practical applications in survey sampling where auxiliary information is available.

Keywords: Double Sampling, Auxiliary Variable, Exponential Estimator, Ratio Estimator, Mean Square Error.

I. INTRODUCTION

In survey sampling, the estimation of population parameters such as the population mean is an important statistical problem. When auxiliary information correlated with the study variable is available, it can be used to improve the precision of estimators. Various estimators such as ratio, regression and product have been developed for this purpose.

Double sampling, also known as two-phase sampling, is commonly used when the auxiliary variable is inexpensive to measure while the study variable is costly or difficult to obtain. In such situations, a large preliminary random sample is drawn and information on the auxiliary variables is collected and a smaller subsample is used to select the study variable.

The use of auxiliary information has been widely studied in survey sampling to increase estimation efficiency. The simplest estimator of population mean is the sample mean estimator

$$\bar{y}_0 = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad 1$$

The mean square error of the estimator $\bar{y}_0$ is given as

$$\text{MSE}(\bar{y}_0) = \bar{Y}\theta_2 C_y^2 \qquad 2$$

which is unbiased but may not be efficient when auxiliary information is available.

The ratio estimator proposed by Cochran (1977) is defined as

$$\bar{y}_C = \bar{y}\left(\frac{\bar{X}\prime}{\bar{x}}\right) \qquad 3$$

which improves efficiency when the study variable is positively correlated with the auxiliary variable.

The exponential ratio estimator proposed by Bahl and Tuteja (1991) is given by

$$\bar{y}_{BT} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \qquad 4$$

which provides improved performance compared with traditional ratio estimators.

Further studies include exponential ratio-product estimators proposed by Chand (1975), Singh and Vishwakarma (2007), and ratio-product estimators proposed by Singh and Espejo (2007).

Singh and Choudhury (2012) developed exponential chain ratio-product estimators using two auxiliary variables. Similarly, Singh and Majhi (2014) proposed chain exponential estimators for two-phase sampling. Other related estimators include those proposed by Khare et al. (2013), Vishwakarma and Kumar (2014), Kadilar (2016), and Faweya *et al.,* 2023 which improved estimation efficiency through modifications of exponential estimators. Although these estimators have shown improved performance, further improvement can be achieved by constructing new estimators that combine exponential ratio estimators with multiple auxiliary variables; hence the proposed estimator.

II. PROPOSED ESTIMATOR

Consider a finite population of size N. Let y be the study variable and x and z be two auxiliary variables. In double sampling, a first-phase sample of size n' is selected to observe the auxiliary variables x and z. A second-phase sample of size n is then selected to observe the study variable y along with the auxiliary variables.

Motivated by existing exponential estimators, a new exponential ratio-type estimator is proposed as

$$\overline{Y}_{AA} = \bar{y}\left[\propto \exp\left(\frac{\bar{x}'}{\bar{x}}\right) + (1-\propto)\exp\left(\frac{\bar{z}'}{\bar{z}} - 1\right)\right] \quad 5$$

Where α is a constant such that $0 \le \alpha \le 1$

To obtain the properties of the proposed estimator, the relative error terms and their expectations are defined as follow:

$$\bar{y} = \overline{Y}(1+\varepsilon_0), \bar{x} = \overline{X}(1+\varepsilon_1), \bar{z} = \overline{Z}(1+\varepsilon_2), \bar{x}' = \overline{X}(1+\varepsilon_1'), \bar{z}' = \overline{Z}(1+\varepsilon_2')$$

Let $\varepsilon_0 = \frac{\bar{y}-\overline{Y}}{\overline{Y}}$, $\varepsilon_1 = \frac{\bar{x}-\overline{X}}{\overline{X}}$, $\varepsilon_1' = \frac{\bar{x}'-\overline{X}}{\overline{X}}$, $\varepsilon_2 = \frac{\bar{z}-\overline{Z}}{\overline{Z}}$, $\varepsilon_2' = \frac{\bar{z}'-\overline{Z}}{\overline{Z}}$

Such that

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_1') = E(\varepsilon_2') = 0$$

And $E(\varepsilon_0^2) = \theta_2 C_y^2$, $E(\varepsilon_1^2) = \theta_2 C_x^2$, $E(\varepsilon_2^2) = \theta_2 C_z^2$, $E(\varepsilon_1'^2) = \theta_1 C_x^2$, $E(\varepsilon_2'^2) = \theta_1 C_z^2$,

$$E(\varepsilon_0\varepsilon_1) = \theta_2 \rho_{yx} C_y C_x$$

$$E(\varepsilon_0\varepsilon_1') = \theta_1 \rho_{yx} C_y C_x,$$

$$E(\varepsilon_1'\varepsilon_1) = \theta_1 C_x^2 \quad E(\varepsilon_0\varepsilon_2) = \theta_2 \rho_{yz} C_y C_z$$

$$E(\varepsilon_0\varepsilon_2') = \theta_1 \rho_{yz} C_y C_z \quad E(\varepsilon_1\varepsilon_2') = \theta_1 \rho_{xz} C_x C_z$$

$$E(\varepsilon_1'\varepsilon_2') = \theta_1 \rho_{xz} C_x C_z \quad E(\varepsilon_1'\varepsilon_2) = \theta_1 \rho_{xz} C_x C_z$$

$$E(\varepsilon_1\varepsilon_2) = \theta_2 \rho_{xz} C_x C_z$$

Where $\theta_1 = \left(\frac{1}{n'} - \frac{1}{N}\right)$, $\theta_2 = \left(\frac{1}{n} - \frac{1}{N}\right)$,

$$k = \frac{n}{n'}$$

$$C_y = \frac{S_y}{\overline{Y}}, \quad C_x = \frac{S_x}{\overline{X}} \quad C_z = \frac{S_z}{\overline{Z}}$$

$$C_{yx} = \frac{\rho_{yx} C_y}{C_x}, \quad C_{yz} = \frac{\rho_{yz} C_y}{C_z},$$

$$C_{zx} = \frac{\rho_{zx} C_x}{C_z}$$

The estimator $\bar{y}_{AA}$ can be expressed in terms of ε 's

$$\bar{y}_{AA} = \overline{Y}(1+\varepsilon_0)\left[\alpha \exp(1+\varepsilon_1')(1+\varepsilon_1)^{-1} + (1-\alpha)\exp(\varepsilon_2' - \varepsilon_2)(1+\varepsilon_2)^{-1}\right] \quad 6$$

Expanding the right hand side of (6) to the second degree of approximation, we have

$$\bar{y}_{AA} = \overline{Y}(1+\varepsilon_0)\left[\alpha \exp(1+\varepsilon_1')(1-\varepsilon_1+\varepsilon_1^2) + (1-\alpha)\exp(\varepsilon_2'-\varepsilon_2)(1-\varepsilon_2+\varepsilon_2^2)\right]$$

$$\bar{y}_{AA} = \overline{Y}(1+\varepsilon_0)\left[\alpha \exp(1-\varepsilon_1+\varepsilon_1^2+\varepsilon_1'-\varepsilon_1'\varepsilon_1) + (1-\alpha)\exp(\varepsilon_2'-\varepsilon_2'\varepsilon_2-\varepsilon_2+\varepsilon_2^2)\right]$$

$$\bar{y}_{AA} - \overline{Y} = \overline{Y}(1+\varepsilon_0)\begin{bmatrix}\alpha\left(\frac{5}{2} - 2\varepsilon_1 + \frac{5\varepsilon_1^2}{2} + 2\varepsilon_1' - \varepsilon_1'\varepsilon_1 - 2\varepsilon_1'\varepsilon_1 + \frac{\varepsilon_1'^2}{2}\right) + \\ (1-\alpha)\left(1+\varepsilon_2' - 2\varepsilon_2'\varepsilon_2 - \varepsilon_2 + \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_2'^2}{2}\right)\end{bmatrix} - \overline{Y}$$

$$\bar{y}_{AA} - \overline{Y} = \overline{Y}\begin{bmatrix}\frac{3\alpha}{2} - 2\alpha\varepsilon_1 + \frac{5\alpha\varepsilon_1^2}{2} + 2\alpha\varepsilon_1' - 3\alpha\varepsilon_1'\varepsilon_1 + \frac{\alpha\varepsilon_1'^2}{2} + 1 + \varepsilon_2' - 2\varepsilon_2'\varepsilon_2 - \varepsilon_2 + \\ \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_2'^2}{2} - \alpha\varepsilon_2' + 2\alpha\varepsilon_2'\varepsilon_2 + \alpha\varepsilon_2 - \frac{3\alpha\varepsilon_2^2}{2} - \frac{\alpha\varepsilon_2'^2}{2} + \frac{5\alpha\varepsilon_0}{2} - 2\alpha\varepsilon_0\varepsilon_1 + \\ 2\alpha\varepsilon_0\varepsilon_1' + \varepsilon_0\varepsilon_2' - \varepsilon_0\varepsilon_2 + \frac{\varepsilon_2'^2}{2} - \alpha\varepsilon_0\varepsilon_2' + \alpha\varepsilon_0\varepsilon_2 + \varepsilon_0\end{bmatrix} - \overline{Y}$$

$$\bar{y}_{AA}-\bar{Y}=\bar{Y}\left[\begin{array}{l}\alpha\left(\begin{array}{l}\frac{3}{2}-2\varepsilon_1+\frac{5\varepsilon_1^2}{2}+2\varepsilon_1'-3\varepsilon_1'\varepsilon_1+\frac{\varepsilon_1'^2}{2}-\varepsilon_2'+2\varepsilon_2'\varepsilon_2+\varepsilon_2\\-\frac{3\varepsilon_2^2}{2}-\frac{\varepsilon_2'^2}{2}+\frac{5\varepsilon_0}{2}-2\varepsilon_0\varepsilon_1+2\varepsilon_0\varepsilon_1'-\varepsilon_0\varepsilon_2{}'+\varepsilon_0\varepsilon_2\end{array}\right)\\+\varepsilon_2'-2\varepsilon_2'\varepsilon_2-\varepsilon_2+\frac{3\varepsilon_2^2}{2}+\frac{\varepsilon_2'^2}{2}+\varepsilon_0\varepsilon_2{}'-\varepsilon_0\varepsilon_2+\frac{\varepsilon_2'^2}{2}+\varepsilon_0\end{array}\right]$$

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Take the expectation of both sides of equation (7) to get the bias of $\bar{y}_{AA}$ in first order approximation

$$E\left(\bar{y}_{AA}-\bar{Y}\right)=\bar{Y}E\left[\begin{array}{l}\alpha\left(\begin{array}{l}\frac{3}{2}-2\varepsilon_1+\frac{5\varepsilon_1^2}{2}+2\varepsilon_1'-3\varepsilon_1'\varepsilon_1+\frac{\varepsilon_1'^2}{2}-\varepsilon_2'+2\varepsilon_2'\varepsilon_2+\varepsilon_2\\-\frac{3\varepsilon_2^2}{2}-\frac{\varepsilon_2'^2}{2}+\frac{5\varepsilon_0}{2}-2\varepsilon_0\varepsilon_1+2\varepsilon_0\varepsilon_1'-\varepsilon_0\varepsilon_2{}'+\varepsilon_0\varepsilon_2\end{array}\right)\\+\varepsilon_2'-2\varepsilon_2'\varepsilon_2-\varepsilon_2+\frac{3\varepsilon_2^2}{2}+\frac{\varepsilon_2'^2}{2}+\varepsilon_0\varepsilon_2{}'-\varepsilon_0\varepsilon_2+\frac{\varepsilon_2'^2}{2}+\varepsilon_0\end{array}\right]$$

$$E\left(\bar{y}_{AA}-\bar{Y}\right)=\bar{Y}\left[\begin{array}{l}\alpha\left(\frac{3}{2}+3\theta_2C_x^2-3\theta_1C_x^2+\frac{3\theta_1C_z^2}{2}-\frac{3\theta_2C_z^2}{2}-2\theta_2\rho_{yx}C_xC_y+2\theta_1\rho_{yx}C_xC_y-\theta_1\rho_{yz}C_yC_z+\theta_2\rho_{yz}C_yC_z\right)+\\\left(\frac{3\theta_2C_z^2}{2}-2\theta_1C_z^2+\theta_1C_z^2+\theta_1\rho_{yz}C_yC_z-\theta_2\rho_{yz}C_yC_z\right)\end{array}\right]$$

Bias $(\bar{y}_{AA})$ =

$$E\left(\bar{y}_{AA}-\bar{Y}\right)=\bar{Y}\left[\begin{array}{l}\alpha\left(\frac{3}{2}+3(\theta_2-\theta_1)C_x^2-\frac{3}{2}(\theta_2-\theta_1)C_z^2-2(\theta_2-\theta_1)\rho_{yx}C_xC_y+(\theta_2-\theta_1)\rho_{yz}C_yC_z\right)+\\\left(\frac{3\theta_2C_z^2}{2}-\theta_1C_z^2-(\theta_2-\theta_1)\rho_{yz}C_yC_z\right)\end{array}\right]$$

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So estimator $\bar{y}_{AA}$ is unbiased if the value of the constant is

$$\alpha=\left\{\frac{(\theta_2-\theta_1)\rho_{yz}C_yC_z+C_z^2\left(\theta_1-\frac{3}{2}\theta_2\right)}{\left(\frac{3}{2}+(\theta_2-\theta_1)\left(3C_x^2-\frac{3}{2}C_z^2-2\rho_{yx}C_xC_y+\rho_{yz}C_yC_z\right)\right)}\right\}$$

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Rewriting equation (7) to the first degree approximation

$$\bar{y}_{AA}-\bar{Y}=\bar{Y}\left[\alpha\left(\frac{3}{2}-2\varepsilon_1+2\varepsilon_1'-\varepsilon_2'+\varepsilon_2+\frac{5\varepsilon_0}{2}\right)+\left(\varepsilon_2'-\varepsilon_2+\varepsilon_0\right)\right]$$

Squaring both sides and neglecting terms of ε's involving power greater than 2

$$\left(\bar{y}_{AA}-\bar{Y}\right)^2=\bar{Y}^2\left[\alpha\left(\frac{3}{2}-2\varepsilon_1+2\varepsilon_1'-\varepsilon_2'+\varepsilon_2+\frac{5\varepsilon_0}{2}\right)+\left(\varepsilon_2'-\varepsilon_2+\varepsilon_0\right)\right]^2$$

$$\left(\bar{y}_{AA}-\bar{Y}\right)^2=\bar{Y}^2\left[\begin{array}{l}\alpha^2\left(\begin{array}{l}\frac{9}{4}-6\varepsilon_1+6\varepsilon_1'-3\varepsilon_2{}'+3\varepsilon_2+\frac{15\varepsilon_0}{2}+4\varepsilon_1^2-8\varepsilon_1'\varepsilon_1+4\varepsilon_1\varepsilon_2'-4\varepsilon_1\varepsilon_2-10\varepsilon_0\varepsilon_1+4\\\varepsilon_1'^2-4\varepsilon_1'\varepsilon_2'+4\varepsilon_1'\varepsilon_2+10\varepsilon_0\varepsilon_1'+\varepsilon_2'-2\varepsilon_2'\varepsilon_2-5\varepsilon_0\varepsilon'+\varepsilon_2^2+5\varepsilon_0\varepsilon_2+\frac{25\varepsilon_0^2}{4}\end{array}\right)\\+\left(\varepsilon_2'^2+\varepsilon_2^2+\varepsilon_0^2-2\varepsilon_2'\varepsilon_2+2\varepsilon_0\varepsilon_2'-2\varepsilon_0\varepsilon_2\right)\\+2\alpha\left(\begin{array}{l}\frac{3\varepsilon_2'}{2}-\frac{3\varepsilon_2}{2}+\frac{3\varepsilon_0}{2}-2\varepsilon_1\varepsilon_2'+2\varepsilon_1\varepsilon_2-2\varepsilon_0\varepsilon_1+2\varepsilon_1'\varepsilon_2'-2\varepsilon_1'\varepsilon_2+2\varepsilon_0\varepsilon_1'-\varepsilon_1'^2+\\\varepsilon_2'\varepsilon_2-\varepsilon_0\varepsilon_2'+\varepsilon_2'\varepsilon_2-\varepsilon_2^2+\varepsilon_0\varepsilon_2+\frac{5}{2}\varepsilon_0\varepsilon_2'-\frac{5}{2}\varepsilon_0\varepsilon_2+\frac{5}{2}\varepsilon_0^2\end{array}\right)\end{array}\right]^2$$

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Taking expectation of both sides of equation 10, the variance of the estimator to first degree of approximation is

$$MSE\left(\bar{y}_{AA}\right)=\bar{Y}^2\left\{\begin{array}{l}\alpha^2\left(\begin{array}{l}\frac{9}{4}+(\theta_2-\theta_1)4C_x^2-(\theta_2-\theta_1)4\rho_{xz}C_xC_z-(\theta_2-\theta_1)10\rho_{xy}C_xC_y+\\(\theta_2-\theta_1)C_z^2+(\theta_2-\theta_1)5\rho_{yz}C_yC_z+\frac{25}{4}\theta_2C_y^2\end{array}\right)\\+(\theta_2-\theta_1)C_z^2+\theta_2C_y^2-(\theta_2-\theta_1)2\rho_{yz}C_yC_z\\+2\alpha\left(\begin{array}{l}(\theta_2-\theta_1)2\rho_{xz}C_xC_z-(\theta_2-\theta_1)2\rho_{xy}C_xC_y-(\theta_2-\theta_1)\frac{3}{2}\rho_{yz}C_yC_z\\-(\theta_2-\theta_1)C_z^2+\frac{5}{2}\theta_2C_y^2\end{array}\right)\end{array}\right\}$$

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The MSE is minimized for

$$\alpha^{opt}=\frac{(\theta_2-\theta_1)\left(2\rho_{yx}C_xC_y-2\rho_{xz}C_xC_z+\frac{3}{2}\rho_{yz}C_yC_z+C_z^2\right)-\frac{5}{2}\theta_2C_y^2}{(\theta_2-\theta_1)\left(4C_x^2-4\rho_{xz}C_xC_z-10\rho_{yx}C_xC_y+C_z^2+5\rho_{yz}C_yC_z\right)+\frac{25}{4}\theta_2C_y^2+\frac{9}{4}}$$

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The MSE of the estimate is:

$$MSE\left(\bar{y}_{AA}\right)=\bar{Y}^2\left\{\theta_2C_y^2+(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)-\frac{\left[(\theta_2-\theta_1)\left(2\rho_{yx}C_xC_y-2\rho_{xz}C_xC_z+\frac{3}{2}\rho_{yz}C_yC_z+C_z^2\right)-\frac{5}{2}\theta_2C_y^2\right]^2}{(\theta_2-\theta_1)\left(4C_x^2-4\rho_{xz}C_xC_z-10\rho_{yx}C_xC_y+C_z^2+5\rho_{yz}C_yC_z\right)+\frac{25}{4}\theta_2C_y^2+\frac{9}{4}}\right\}$$

$$A=(\theta_2-\theta_1)\left(2\rho_{yx}C_xC_y-2\rho_{xz}C_xC_z+\frac{3}{2}\rho_{yz}C_yC_z+C_z^2\right)-\frac{5}{2}\theta_2C_y^2$$

$$B=(\theta_2-\theta_1)\left(4C_x^2-4\rho_{xz}C_xC_z-10\rho_{yx}C_xC_y+C_z^2+5\rho_{yz}C_yC_z\right)+\frac{25}{4}\theta_2C_y^2+\frac{9}{4}$$

Then,

$$MSE[\bar{Y}_{AA}]^{opt}=$$
$$\bar{Y}^2\left\{\theta_2C_y^2+[\theta_2-\theta_1]C_z^2[1-2C_{yz}]-\frac{[A]^2}{B}\right\}$$

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III. MATHEMATICAL COMPARISON

To compare the efficiency, some conditions are obtained by comparing the mean square errors of the estimators under which the proposed estimator performs better than the other existing estimators.

Comparison of the proposed estimator with sample mean per unit $\bar{y}_0$ estimator

$$MSE\left(\bar{y}_0\right)-MSE\left(\bar{y}_{AA}\right)=\bar{Y}^2\left\{\frac{A^2}{B}-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)\right\}$$

Thus the proposed estimator $\bar{y}_{AA}$ will be efficient over the estimator $\bar{y}_0$ if

$$\left\{\frac{A^2}{B}-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)\right\}\geq 0$$

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Comparison of the proposed estimator with Singh and Choudhury (2012) $\bar{y}_{SC}$ estimator

$$MSE\left(\bar{y}_{SC}\right)-MSE\left(\bar{y}_{AA}\right)=\bar{Y}^{2}\left\{(\theta_2-\theta_1)C_x^2\left(\tfrac{1}{4}-C_{yx}\right)+\theta_1 C_z^2\left(\tfrac{1}{4}-C_{yz}\right)-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)+\frac{A^2}{B}\right\}\geq 0$$

Thus the proposed estimator $\bar{y}_{AA}$ will be efficient over the estimator $\bar{y}_{SC}$ if

$$\left\{(\theta_2-\theta_1)C_x^2\left(\tfrac{1}{4}-C_{yx}\right)+\theta_1 C_z^2\left(\tfrac{1}{4}-C_{yz}\right)-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)+\frac{A^2}{B}\right\}\geq 0$$

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Comparison of the proposed estimator with Vishwakarma and Kumar (2014) $\bar{y}_{VK}$ estimator

$$MSE\left(\bar{y}_{VK}\right)-MSE\left(\bar{y}_{AA}\right)=\bar{Y}^{2}\left\{\frac{A^2}{B}-\frac{\left((\theta_2-\theta_1)\rho_{xy}C_xC_y+\theta_1\rho_{yz}C_yC_z\right)^2}{(\theta_2-\theta_1)C_x^2+\theta_1C_z^2}-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)\right\}\geq 0$$

Thus the proposed estimator $\bar{y}_{AA}$ will be efficient over the estimator $\bar{y}_{VK}$ if

$$\left\{\frac{A^2}{B}-\frac{\left((\theta_2-\theta_1)\rho_{xy}C_xC_y+\theta_1\rho_{yz}C_yC_z\right)^2}{(\theta_2-\theta_1)C_x^2+\theta_1C_z^2}-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)\right\}\geq 0$$

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Comparison of the proposed estimator with Singh and Ahmed (2015) $\bar{y}_{SA}$ estimator

$$MSE\left(\bar{y}_{SA}\right)-MSE\left(\bar{y}_{AA}\right)=\bar{Y}^{2}\left\{(\theta_2-\theta_1)C_x^2\left(\tfrac{1}{8}+C_{yx}\right)+\theta_1 C_z^2\left(\tfrac{1}{8}+C_{yz}\right)-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)+\frac{A^2}{B}\right\}\geq 0$$

Thus the proposed estimator $\bar{y}_{AA}$ will be efficient over the estimator $\bar{y}_{SA}$ if

$$\left\{(\theta_2-\theta_1)C_x^2\left(\tfrac{1}{8}+C_{yx}\right)+\theta_1 C_z^2\left(\tfrac{1}{8}+C_{yz}\right)-(\theta_2-\theta_1)C_z^2\left(1-2C_{yz}\right)+\frac{A^2}{B}\right\}\geq 0$$

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IV. EMPIRICAL STUDY

The empirical results were obtained using data collected from 1,023 workers in Akure South Local Government Area. The double sampling uses tax as the study variable y, grade level as the auxiliary variable x and gross payment as auxiliary variable z.

Table 1 Summary of the mean, MSE, CV and RE of each estimator

Estimator	$\bar{y}_{AA}$	$\bar{y}_{SA}$	$\bar{y}_{VK}$	$\bar{y}_{SC}$	$\bar{y}_0$
MSE	126091.24	415290.46	244536.41	273046.79	361020.74
CV	5.52	9.49	7.49	7.71	8.78
RE	286.32	86.93	147.64	132.22	100

The results indicate proposed estimator is the most efficient estimator among all the estimators considered. It has the lowest MSE (126091.24) and lowest CV (5.52%), indicating higher precision and stability compared with the others. Its Relative Efficiency of 286.32% shows that it performs almost three times better than the conventional estimator.

V. CONCLUSION

This study proposed a new exponential ratio-type estimator for estimating the population mean in double sampling using two auxiliary variables. The bias and mean square error of the estimator were derived and the optimum value of the constant involved was obtained.

Both theoretical and empirical comparisons demonstrate that the proposed estimator performs better than several existing estimators in terms of efficiency. The empirical results showed that the estimator has the smallest mean square error and the highest relative efficiency.

Therefore, the proposed estimator is recommended for practical applications in survey sampling where auxiliary information is available.

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