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Modeling Inflation Dynamics with Atangana-Baleanu Fractional Derivatives: A Memory-Driven Approach to Consumer Price Index Forecasting

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Abstract- Inflation dynamics are widely characterized by persistence, delayed adjustment, and pronounced uncertainty, features that are often inadequately captured by conventional integer-order models. This paper develops and applies an Atangana-Baleanu fractional-order inflation framework to analyze Consumer Price Index (CPI) forecast dynamics, with particular emphasis on memory effects, expectation formation, and forecast uncertainty. By incorporating a non-singular kernel and fractional memory, the proposed model allows inflation deviations to decay according to a Mittag-Leffler type process rather than standard exponential adjustment. Using CPI forecast data over multiple horizons, the empirical analysis demonstrates that lower fractional orders are associated with stronger inflation persistence, slower convergence to long-run equilibrium, and sustained forecast dispersion. Comparative evaluation against classical integer-order benchmark models reveals that the fractional framework achieves superior forecasting performance, as reflected in lower error metrics and reduced serial correlation in residuals, especially during periods of elevated macroeconomic uncertainty. The results further suggest that inflation expectations are formed through adaptive and history-dependent mechanisms, with past shocks exerting prolonged influence on both the level and uncertainty of forecasts. On the whole, the findings highlight the economic relevance of the fractional-order parameter as a meaningful measure of inflation inertia and provide strong support for the use of Atangana-Baleanu fractional models in empirical inflation analysis and macroeconomic forecasting.

Index Terms- Fractional calculus; Atangana-Baleanu derivative; Inflation dynamics; CPI forecasting; Long memory.

I. INTRODUCTION

1.1 Background and Motivation

Inflation dynamics play a central role in macroeconomic stability, monetary policy formulation, and economic planning. As a primary indicator of inflation, the Consumer Price Index (CPI) is widely used by policymakers, central banks, and financial institutions to assess changes in the cost of living and to guide interest rate decisions, wage negotiations, and index-linked contracts. Accurate CPI forecasting is therefore essential for managing inflation expectations, ensuring price stability, and minimizing macroeconomic uncertainty [1, 2]. Persistent inflationary pressures, particularly following economic shocks, further underscore the need for models that can adequately represent the temporal dependence inherent in inflation processes. Earlier studies have formulated Consumer Price Index (CPI) dynamics using linear modeling frameworks, which typically result in systems of linear equations. The efficient solution of such systems is crucial, and iterative techniques like the Jacobi and Gauss–Seidel methods are frequently adopted. A comparative analysis of these methods, particularly in terms of convergence behavior and computational efficiency, has been presented in [3].

Traditional CPI forecasting methods are predominantly based on integer-order time-series and differential models, including autoregressive integrated moving average (ARIMA), vector autoregression (VAR), and state-space frameworks. While these approaches have proven effective for short-term prediction under stable conditions, they often struggle to capture the long-range dependence and persistence observed in inflation and inflation expectations [4,5]. Integer-order models typically assume exponentially decaying memory, which limits their ability to reflect the cumulative influence of historical shocks and structural changes on current inflation dynamics.

Empirical studies have consistently demonstrated that inflation exhibits long-memory behavior, characterized by slow decay of autocorrelations and fractional integration properties. Seminal work by [6, 7] introduced fractional differencing as a means of modeling such persistence, while subsequent studies confirmed that inflation and inflation expectations often follow fractionally integrated processes rather than purely stationary or unit-root dynamics [8, 9]. These findings suggest that inflation responds to past disturbances over extended time horizons, challenging the assumptions underlying conventional integer-order models.

More recent empirical evidence has reinforced the relevance of long-memory effects in inflation forecasting. Studies examining survey-based inflation expectations and core CPI components indicate that persistence varies across regimes and economic conditions, with memory effects intensifying during periods of heightened uncertainty [10,11]. Such behavior motivates the adoption of modeling frameworks that explicitly incorporate non-local and history-dependent dynamics.

Fractional calculus provides a mathematically rigorous framework for capturing long memory effects through non-integer-order differentiation. Unlike classical derivatives, fractional derivatives embed historical information via convolution kernels, allowing present dynamics to depend on the entire past trajectory of the system [12]. In this context, fractional differential equations offer a natural extension of traditional inflation models, enabling more flexible representations of persistence and adjustment speed.

The Atangana-Baleanu fractional derivative, characterized by a nonsingular Mittag-Leffler kernel, represents a significant advancement over classical fractional operators. Its ability to model memory without singular behavior makes it particularly well-suited for economic systems where smooth temporal transitions and bounded memory effects are desirable [13]. These properties motivate the present study's focus on developing a memory-driven CPI forecasting framework grounded in Atangana-Baleanu fractional dynamics.

1.2 Fractional Calculus in Economic Modeling

Classical economic and financial models are traditionally formulated using integer-order differential or difference equations, which implicitly assume short memory and exponential decay of past information. While these assumptions simplify mathematical treatment, they often fail to capture empirically observed features of economic systems such as persistence, long-range dependence, and slow adjustment dynamics. In particular, macroeconomic variables including inflation, output, interest rates, and volatility frequently exhibit memory effects that extend far beyond the scope of standard autoregressive structures [5, 6].

Fractional calculus provides a natural extension of classical modeling frameworks by allowing derivatives and integrals of non-integer order. Fractional derivatives generalize the notion of differentiation by embedding memory directly into the system dynamics [14], enabling current states to depend on the entire historical trajectory of a variable rather than on a finite number of past observations [12, 15]. This property makes fractional calculus particularly well suited for economic and financial systems, where agents decisions are shaped by accumulated experience, expectations, and delayed responses to shocks.

In financial economics, fractional models have been successfully employed to describe asset price dynamics, volatility clustering, and anomalous diffusion processes that deviate from classical Brownian motion assumptions [16, 17]. Similarly, in macroeconomics, fractional integration and long-memory processes have been shown to improve the modeling of persistent inflation and output fluctuations, offering a more flexible representation than standard ARIMA or state-space models [4, 9]. These applications highlight the ability of fractional-order models to bridge the gap between stationary and unit-root behavior commonly observed in

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economic time series. A key advantage of fractional calculus lies in the use of non-local operators, which explicitly account for historical dependence [18]. Unlike local integer-order derivatives that rely solely on instantaneous rates of change, fractional derivatives incorporate weighted contributions from past states, with the weighting structure governed by the fractional order. This non-locality allows models to represent gradual information diffusion, expectation anchoring, and inertia in economic adjustment processes [7, 5].

Recent advances in fractional calculus have further enhanced its applicability to economic modeling by introducing derivatives with non-singular kernels. Traditional fractional derivatives with power-law kernels may impose infinite memory at the origin, which can be difficult to reconcile with economic interpretation. In contrast, modern formulations based on Mittag-Leffler kernels provide bounded and smoothly decaying memory, offering a more realistic description of how past economic information influences present behavior [13, 19]. These developments make fractional calculus a powerful and interpretable tool for modeling historical dependence in economic and financial systems.

In general, the integration of fractional calculus into economic modeling enables a richer and more flexible representation of persistence, memory, and delayed adjustment. By moving beyond the restrictive assumptions of integer-order dynamics, fractional-order frameworks offer improved explanatory power and forecast performance in environments characterized by long-memory behavior and structural complexity.

1.3 Atangana-Baleanu Fractional Derivative

Recent developments in fractional calculus have introduced a new class of fractional derivatives designed to overcome conceptual and mathematical limitations associated with classical formulations. Among these, the Atangana-Baleanu (AB) fractional derivative has attracted significant attention due to its non-singular and non-local kernel based on the Mittag-Leffler function [20]. This formulation represents a fundamental departure from traditional Riemann-Liouville and Caputo derivatives, which rely on singular power-law kernels that impose infinite memory at the initial time [12, 15].

The Atangana-Baleanu fractional derivative is characterized by a non-singular kernel, ensuring that the memory contribution remains bounded for all past times [21]. This property enhances numerical stability and facilitates more realistic interpretations of memory in physical and socio-economic systems [13]. The absence of kernel singularity eliminates abrupt weighting of initial conditions and allows smooth integration of historical information into system dynamics. Non-locality is a defining feature of the Atangana-Baleanu operator. Unlike integer-order derivatives, which depend solely on local information, the AB derivative incorporates the entire history of the state variable through an integral formulation. The weighting of past states is governed by a Mittag-Leffler function, which generalizes exponential decay and allows memory to diminish gradually rather than abruptly [17, 19]. This structure enables the modeling of systems where historical dependence plays a persistent but bounded role. The Mittag-Leffler memory structure embedded in the Atangana-Baleanu derivative offers a flexible representation of memory effects. Compared to power-law memory, Mittag-Leffler decay captures intermediate behavior between exponential and algebraic decay, making it particularly suitable for systems exhibiting long but finite memory [22, 23]. This feature has proven advantageous in modeling diffusion, viscoelasticity, biological dynamics, and complex adaptive systems [24, 25]. From a modeling perspective, the Atangana-Baleanu operator is well-suited for inflation dynamics and CPI forecasting. Empirical studies consistently document persistence and inertia in inflation and inflation expectations, with shocks dissipating slowly over time [4, 5]. Classical integer-order models often impose exponential adjustment, which may underestimate the degree of persistence observed in practice. By contrast, the non-local and memory-driven structure of the AB derivative allows inflation dynamics to reflect gradual adjustment and historical dependence in a mathematically consistent manner. Moreover, the bounded memory implied by the nonsingular kernel aligns closely with economic intuition. Inflation expectations are influenced by past outcomes, but their impact diminishes smoothly as agents update

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beliefs based on new information. The Atangana-Baleanu formulation captures this behavior without assuming infinite memory or unrealistic persistence [16, 17]. Consequently, the AB derivative provides a robust and interpretable framework for modeling inflation processes characterized by long memory, uncertainty, and delayed policy transmission.

In summary, the Atangana-Baleanu fractional derivative offers a mathematically sound and economically meaningful tool for modeling inflation dynamics. Its key properties, i.e. non-singular kernel, non-locality, and Mittag-Leffler memory, make it particularly well suited for capturing persistence and historical dependence in CPI forecasting and other macroeconomic applications.

1.4 Research Objectives and Contributions

The primary objective of this study is to develop a robust and memory-driven framework for modeling inflation dynamics by leveraging the Atangana-Baleanu fractional derivative. By integrating fractional calculus into CPI forecasting, the study aims to overcome the limitations of classical integer-order models in capturing persistence, historical dependence, and smooth adjustment behavior observed in inflation processes.

Specifically, the first objective is to formulate a fractional-order inflation model using the Atangana-Baleanu derivative in the Caputo sense. The proposed model incorporates non-local memory effects through a nonsingular Mittag-Leffler kernel, enabling a flexible representation of inflation persistence and expectation dynamics. This formulation provides a mathematically consistent extension of conventional inflation models while preserving economic interpretability.

The second objective is to conduct a rigorous theoretical analysis of the proposed fractional model. This includes establishing conditions for the existence and uniqueness of solutions to the resulting fractional differential equations, as well as analyzing their stability properties. In particular, the study investigates Mittag-Leffler stability to characterize the long-term behavior of inflation dynamics under different fractional orders and parameter settings.

The third objective is to develop and implement an appropriate numerical solution scheme for the Atangana-Baleanu fractional inflation model. Using this scheme, the model is applied to CPI forecast data to evaluate its ability to reproduce observed inflation trajectories and memory effects. Numerical simulations are used to examine the sensitivity of inflation dynamics to fractional order and to assess the practical feasibility of the proposed approach.

Finally, the study provides a comparative evaluation of the fractional-order model against classical integer-order modeling approaches commonly used in inflation forecasting. By contrasting forecasting behavior, persistence representation, and dynamic adjustment properties, the paper offers insights into the advantages of fractional dynamics for capturing long-memory inflation processes. These comparisons highlight the potential of the Atangana-Baleanu framework as a viable alternative for advanced CPI forecasting and inflation analysis.

The contributions of this paper are threefold: (i) the introduction of a novel fractional order inflation model based on the Atangana-Baleanu derivative, (ii) the provision of theoretical guarantees regarding solution behavior and stability, and (iii) the demonstration of improved dynamic representation of inflation persistence through numerical and empirical analysis.

II. LITERATURE REVIEW

2.1 Classical Inflation and CPI Forecasting Models

Classical approaches to modeling inflation and forecasting the Consumer Price Index (CPI) are predominantly grounded in linear time-series and econometric frameworks. Among the most widely used models are autoregressive integrated moving average (ARIMA) models, vector autoregression (VAR), state-space representations, and structural econometric models. These methodologies form the foundation of many central bank forecasting systems and have been extensively applied due to their interpretability, statistical rigor, and ease of implementation [1].

ARIMA models represent inflation as a univariate stochastic process characterized by autoregressive and moving average components, with differencing used to address non-stationarity. These models are particularly effective for short-term forecasting when inflation dynamics are relatively stable and well captured by historical patterns. However, ARIMA models assume exponentially decaying memory and linear adjustment, which limits their ability to represent persistent inflation dynamics and structural breaks commonly observed in CPI data [26].

Vector autoregressive (VAR) models extend univariate approaches by jointly modeling inflation with other macroeconomic variables such as output, interest rates, and exchange rates. VAR frameworks allow for rich dynamic interactions and impulse response analysis, making them useful for policy evaluation and scenario analysis. Despite these advantages, VAR models often require large datasets for reliable estimation and remain constrained by short-memory assumptions, which can lead to poor performance in the presence of long-range dependence and evolving inflation regimes [4].

State-space models and their associated filtering techniques, such as the Kalman filter, offer a flexible framework for modeling time-varying inflation dynamics and unobserved components. These models are widely used to decompose inflation into trend and cyclical components and to account for stochastic volatility. While state-space representations improve adaptability and uncertainty quantification, they typically rely on linear Gaussian assumptions and local dynamics, which may inadequately capture the persistent influence of past shocks and expectations on inflation [27].

Structural econometric models incorporate economic theory directly into inflation modeling by linking price dynamics to fundamentals such as monetary policy, labor markets, and supply-side factors. Although these models provide valuable economic interpretation, they are often sensitive to model specification and parameter stability. Moreover, their reliance on integer-order dynamics constrains their capacity to represent gradual adjustment and long-memory behavior, particularly during periods of heightened uncertainty or regime change [2]

Summarily, while classical inflation and CPI forecasting models offer important insights and practical forecasting tools, their underlying short-memory assumptions and local dynamics limit their effectiveness in capturing persistence and uncertainty. These limitations motivate the exploration of alternative modeling approaches, such as fractional-order and non-local frameworks, which can more accurately represent the memory-driven nature of inflation processes.

2.2 Long-Memory and Fractional Models in Economics

Empirical research in macroeconomics and finance has consistently shown that many economic time series exhibit long-memory behavior, characterized by persistent dependence between observations far apart in time. Long-range dependence is typically identified through slowly decaying autocorrelation functions and spectral densities that diverge at low frequencies. Such properties contradict the short-memory assumptions underlying classical autoregressive and moving-average models and motivate the use of fractional modeling frameworks [6].

Fractional integration provides a flexible approach for modeling long-memory processes by allowing the differencing parameter to take non-integer values. In fractionally integrated models, such as ARFIMA processes, the degree of persistence is governed by a fractional order parameter that interpolates between stationary and unit-root behavior. This framework has been widely applied to macroeconomic variables including inflation, output growth, interest rates, and exchange rates, where persistence often lies between short-run fluctuations and permanent shocks [5]. Empirical evidence suggests that inflation and CPI series are frequently better described by fractional integration than by purely stationary or integrated models. Beyond fractional differencing in discrete-time econometrics, fractional differential equations (FDEs) have emerged as a continuous-time alternative for modeling economic dynamics with memory. Fractional derivatives generalize classical derivatives by incorporating nonlocal operators that depend on the entire historical path of a variable. This feature enables FDEs to represent gradual adjustment processes, learning effects, and persistent responses to shocks that are commonly observed in economic and financial systems [28]. As a result, fractional calculus has been increasingly adopted in the modeling of asset prices, volatility, interest rate dynamics, and macroeconomic adjustment mechanisms.

Applications of fractional differential equations in finance have demonstrated their effectiveness in capturing anomalous diffusion, volatility clustering, and long-term dependence in financial markets. Fractional models have been used to describe option pricing dynamics, interest rate evolution, and risk propagation, offering improved flexibility over classical stochastic differential equations [29]. These applications highlight the potential of fractional calculus to unify empirical long-memory observations with continuous-time dynamic modeling.

In economics, the adoption of fractional differential equations remains relatively limited but growing. Existing studies suggest that fractional dynamic models can enhance the representation of persistence and inertia in macroeconomic systems, particularly in contexts where policy effects unfold gradually over time. The integration of fractional calculus into inflation and CPI modeling therefore represents a natural extension of the long-memory literature, providing a continuous-time, memory-driven framework that complements traditional econometric approaches and addresses their limitations in capturing long-range dependence.

2.3 Atangana-Baleanu Fractional Operators

The Atangana-Baleanu (AB) fractional operators represent a significant advancement in the theory of fractional calculus, addressing several mathematical limitations associated with classical fractional derivatives. Introduced by Atangana and Baleanu, these operators are defined using a non-singular and non-local kernel based on the Mittag-Leffler function, which ensures bounded memory and smooth temporal behavior [13]. Unlike traditional Riemann-Liouville and Caputo derivatives, whose kernels exhibit singular power-law behavior, the Atangana-Baleanu formulation avoids divergence at the origin and provides improved stability and interpretability. From a theoretical standpoint, the Atangana-Baleanu fractional derivative has been developed in both Caputo and Riemann senses, allowing flexibility in initial condition specification and analytical treatment. The Mittag-Leffler kernel embedded in the operator generalizes exponential decay and enables the modeling of memory effects with tunable persistence controlled by the fractional-order parameter. This formulation has been shown to possess well-defined Laplace transforms, facilitating analytical solution techniques and stability analysis for fractional differential equations [15, 30]. The mathematical robustness of the Atangana-Baleanu operator has led to its widespread adoption in modeling complex systems across multiple scientific domains. In physical sciences, AB fractional derivatives have been successfully applied to heat transfer, diffusion processes, and viscoelastic materials, where memory effects and nonlocal interactions play a critical role [13]. In biological systems, the operator has been used to model population dynamics, epidemiological processes, and biological transport mechanisms, capturing hereditary effects and delayed responses more accurately than classical models [19].

More recently, applications of Atangana-Baleanu operators have extended to socio-economic systems, including models of financial market dynamics, risk propagation, and decision-making processes. These studies highlight the operator's capacity to represent cumulative behavioral effects, adaptive learning, and persistent shocks features that are highly relevant for economic modeling [17]. The nonsingular memory structure ensures that long-term dependence is incorporated without imposing unrealistic infinite-memory assumptions, making the operator suitable for real-world socio-economic phenomena. Despite these advances, the application of Atangana-Baleanu fractional operators to inflation dynamics and CPI forecasting remains largely unexplored. Existing economic studies on long memory primarily rely on fractional integration in discrete-time econometrics or classical fractional differential equations with singular kernels. There is a clear research gap in developing continuous time inflation models that leverage the non-singular, non-local properties of the Atangana-Baleanu derivative. Addressing this gap offers the potential to improve the representation of inflation persistence, expectation formation, and gradual policy transmission, thereby motivating the present study's focus on Atangana-Baleanu based CPI modeling.

2.4 Identified Research Gap

The reviewed literature reveals substantial progress in inflation and CPI modeling using classical econometric frameworks, long-memory time-series techniques, and fractional integration approaches. While these methods have improved the understanding of inflation persistence and uncertainty, they remain largely confined to discrete-time settings and rely on either short-memory assumptions or fractional differencing mechanisms that do not fully exploit the capabilities of continuous-time, non-local modeling frameworks. Existing applications of fractional calculus in economics and finance have primarily focused on classical fractional derivatives with singular kernels or on ARFIMA-type models that capture long-range dependence through fractional integration. Although these approaches successfully identify persistence in macroeconomic variables, they are limited in their ability to represent smooth adjustment dynamics and bounded memory effects, which are empirically relevant for inflation and expectation formation. Moreover, singular-kernel fractional derivatives may introduce mathematical and interpretational challenges that are not well aligned with economic behavior. Recent theoretical developments surrounding the Atangana-Baleanu fractional derivative provide a promising alternative due to their non-singular and non-local memory structure. Despite successful applications of Atangana-Baleanu operators in physical, biological, and complex-system modeling, their use in macroeconomic applications particularly inflation dynamics and CPI forecasting remains notably absent from literature. To date, no comprehensive framework has been established that integrates Atangana-Baleanu fractional operators into inflation modeling with rigorous theoretical analysis and empirical validation. Furthermore, comparative assessments between Atangana-Baleanu based fractional models and classical integer-order inflation models are scarce. As a result, the potential benefits of non-singular fractional memory in capturing inflation persistence, uncertainty, and gradual policy transmission have not been systematically evaluated. This gap limits the advancement of memory-driven modeling approaches in macroeconomic forecasting.

In response to these gaps, the present study proposes a novel fractional-order inflation model based on the Atangana-Baleanu derivative. By combining rigorous mathematical analysis, numerical simulation, and application to CPI forecast data, the study aims to bridge the disconnect between advanced fractional calculus theory and practical inflation modeling. This contribution addresses an unmet need in literature for continuous-time, non-singular, and non-local approaches to modeling inflation dynamics.

III. METHODOLOGY

3.1 Data Description and Preprocessing

The empirical analysis in this study is based on a Consumer Price Index (CPI) forecast dataset designed to capture both expected inflation dynamics and forecast uncertainty. rather than relying solely on realized CPI inflation, the dataset reports forward-looking inflation projections in the form of lower, mid, and upper forecast bounds. Such forecast distributions are commonly produced by central banks and macroeconomic forecasting institutions to reflect alternative inflation scenarios and uncertainty surrounding future price developments [1, 2].

The mid forecast represents the central or most-likely inflation trajectory and serves as the primary series for model calibration. The lower and upper bounds correspond to pessimistic and optimistic inflation scenarios, respectively, and jointly define a forecast envelope that reflects uncertainty in inflation expectations. This structure is particularly well suited for studying inflation persistence and expectation formation, as forecast revisions typically evolve gradually across successive forecast vintages rather than adjusting instantaneously [4, 5]. The availability of forecast bounds allows the analysis to assess not only central tendency but also the persistence of inflation uncertainty over time. The CPI forecast series is observed at discrete and equally spaced time intervals corresponding to forecast release dates. To facilitate fractional-order modeling in continuous time, the data is mapped onto a uniform temporal grid. Let $t_n=n\Delta t$, where Δt denotes the fixed time step and n=0,1,…,N. This discretization enables numerical approximation of the underlying fractional differential equations while preserving the temporal ordering of forecast information [12, 15]. Discretized time representation is essential for implementing predictor-corrector schemes used to solve fractional models. Prior to estimation, the CPI forecast series undergoes normalization to improve numerical stability and comparability across different forecast horizons. Specifically, each forecast series is standardized using

x_n^*=(x_n-μ)/σ,

where x_n denotes the original CPI forecast value at time t_n , and μ , σ are the sample mean and standard deviation, respectively. Standardization ensures that the scale of the data does not distort the estimation of model parameters or the numerical solution of fractional differential equations [26]. This preprocessing step is particularly important for fractional systems, where memory effects can amplify numerical instability if the data is poorly scaled. In addition, the dataset is screened for missing observations and inconsistencies across forecast bounds. Any missing values are treated using interpolation methods consistent with the smooth evolution of forecast paths, while extreme observations are evaluated relative to the forecast envelope. These preprocessing steps ensure that the CPI forecast dataset is well conditioned for fractional-order modeling and suitable for analyzing memory-driven inflation dynamics.

3.2 Model Formulation

This section presents the mathematical formulation of the fractional-order inflation model based on the Atangana-Baleanu fractional derivative in the Caputo sense. The formulation integrates non-local memory effects into inflation dynamics while preserving well-defined initial conditions and economic interpretability.

3.2.1 Atangana-Baleanu Fractional Derivative in the Caputo Sense

Let $x(t)$ be a sufficiently smooth function defined on $[0,T]$. The Atangana-Baleanu fractional derivative in the Caputo sense of order $\alpha \in (0,1]$ is defined as

$$ {}^{ABC}D_t^{\alpha} x(t) = \frac{B(\alpha)}{1-\alpha} \int_0^t x'(s) E_\alpha \left(-\frac{\alpha}{1-\alpha} (t-s)^\alpha \right) ds \quad (1)$$

where $E_\alpha(\cdot)$ denotes the Mittag-Leffler function, and $B(\alpha)$ is a normalization constant satisfying $B(0) = B(1) = 1$. The Mittag-Leffler kernel ensures a non-singular and non-local memory structure, allowing the derivative to account for the entire historical trajectory of the system while avoiding singular behavior at the initial time.

3.2.2 Fractional Inflation Dynamic Equation

Let $x(t)$ represent the normalized CPI inflation forecast at time t. The inflation dynamics aremmodeled using a fractional differential equation of the form

$$ {}^{ABC}D_t^{\alpha} x(t) = \lambda(x(t) - \bar{x}) + u(t), \quad (2)$$

Subject to the initial condition,

$$x(0) = x_0 \quad (3)$$

where $\lambda \in \mathbb{R}$ denotes the adjustment or mean-reversion coefficient, $\bar{x}$ represents the long-run equilibrium inflation level, and $u(t)$ captures exogenous influences such as policy interventions, demand shocks, or expectation adjustments. Equation (2) generalizes classical first-order inflation models by replacing the integer-order time derivative with the Atangana-Baleanu fractional derivative. This formulation allows inflation dynamics to evolve under the influence of both current conditions and accumulated historical effects, consistent with empirical observations of persistent inflation behavior.

3.2.3 Interpretation of Model Parameters and Fractional Order

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The parameter λ governs the speed and direction of adjustment toward the long-run inflation level. Negative values of λ correspond to mean-reverting behavior, while values closer to zero imply slower adjustment and greater persistence. The equilibrium level $\bar{x}$ reflects the target or steady-state inflation rate implied by long-term economic conditions or policy objectives. The fractional order α plays a central role in determining the memory characteristics of the inflation process. When $\alpha = 1$, the model reduces to a classical integer-order differential equation with local dynamics and exponential memory decay. For $0 < \alpha < 1$, the system exhibits long memory behavior, with past inflation outcomes exerting a persistent influence on current dynamics through the Mittag-Leffler kernel. Smaller values of α correspond to stronger memory effects and slower decay of historical influence. By jointly estimating $\lambda, \bar{x}, and\ \alpha,$ the proposed framework enables a flexible representation of inflation persistence, adjustment speed, and historical dependence. This structure is particularly well-suited for modeling CPI forecast dynamics, where inflation expectations evolve gradually and reflect the cumulative impact of past economic conditions and policy actions.

3.3 Mathematical Analysis

The use of Atangana–Baleanu fractional derivatives provides a robust framework for modeling memory-dependent inflation dynamics and Consumer Price Index (CPI) evolution. Analyzing such systems requires effective tools for stability and threshold determination. In related nonlinear settings, the next generation matrix approach has been used to derive key thresholds such as the basic reproduction number [31], emphasizing the role of spectral properties. These concepts extend naturally to fractional-order economic models. This section investigates the analytical properties of the proposed fractional inflation model. In particular, the existence and uniqueness of solutions are established, followed by a stability analysis based on Mittag-Leffler functions. The role of the fractional order in shaping inflation dynamics is also examined.

3.3.1 Existence and Uniqueness of Solutions

Consider the fractional inflation model defined by

$$ {}^{ABC}D_t^{\alpha} x(t) = \lambda(x(t) - \bar{x}) + u(t), \quad x(0) = x_0 \tag{4}$$

where $0 < \alpha \leq 1$, $\lambda \in \mathbb{R}$ and $u(t)$ is a continuous function on $[0, T]$. Equation (4) can be rewritten in integral form using the properties of the Atangana-Baleanu fractional derivative as

$$x(t) = x_0 + \frac{1-\alpha}{B(\alpha)}[\lambda(x(t) - \bar{x}) + u(t)] + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[\lambda(x(s) - \bar{x}) + u(s)]ds \tag{5}$$

Define the operator Υ on the Banach space $C([0, \Upsilon], \mathbb{R})$ by

$$(\Upsilon x)(t) = x_0 + \frac{1-\alpha}{B(\alpha)}[\lambda(x(t) - \bar{x}) + u(t)] + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}[\lambda(x(s) - \bar{x}) + u(s)]ds.$$

Under the assumption that the right-hand side of (4) is Lipschitz continuous with respect to x, the operator Υ is a contraction for sufficiently small T. Hence, by Banach's fixed-point theorem, there exists a unique continuous solution $x(t)$ on $[0, T]$. This establishes the local existence and uniqueness of solutions to the Atangana-Baleanu fractional inflation model.

3.3.2 Mittag-Leffler Stability Analysis

To analyze stability, consider the homogeneous version of the model

$$ {}^{ABC}D_t^{\alpha} x(t) = \lambda(x(t) - \bar{x}) \tag{6}$$

Let $y(t) = x(t) - \bar{x}$. Then (6) becomes

$$ {}^{ABC}D_t^{\alpha} x(t) = \lambda y(t) \tag{7}$$

The solution of (7) is given by

$$y(t) = y(0)E_{\alpha}\left(\frac{\lambda\alpha}{B(\alpha)}t^{\alpha}\right), \tag{8}$$

where $E_{\alpha}(\cdot)$ denotes the Mittag-Leffler function.

The equilibrium point $x = \bar{x}$ is said to be Mittag-Leffler stable if

$$|y(t)| \leq CE_{\alpha}(-\kappa t^{\alpha}), \kappa > 0,$$

for all $t \geq 0$. When $\lambda < 0$ the argument of the Mittag-Leffler function is negative, and the solution decays asymptotically to zero. Hence, the equilibrium inflation level $\bar{x}$ is Mittag-Leffler stable, indicating that inflation converges smoothly toward its long-run level under fractional dynamics.

3.3.3 Influence of the Fractional Order on System Behavior

The fractional order α determines the strength of memory effects in the inflation process. For $\alpha = 1$, the Atangana-Baleanu derivative is reduced to the classical first-order derivative, and the system exhibits exponential convergence to equilibrium. As α decreases below unity, the decay governed by the Mittag-Leffler function becomes slower, reflecting stronger persistence and long memory effects.

Lower values of α imply that past inflation deviations exert a more prolonged influence on current dynamics, resulting in gradual adjustment and delayed convergence. This behavior is consistent with empirical observations of inflation inertia and expectation anchoring. Consequently, the fractional order acts as a quantitative measure of inflation persistence, with smaller α corresponding to higher degrees of memory and slower policy transmission effects. Furthermore, the mathematical analysis confirms that the proposed Atangana-Baleanu fractional inflation model admits a unique solution, exhibits Mittag-Leffler stability, and allows flexible control of persistence through the fractional order parameter.

3.4 Numerical Solution Scheme

Due to the non-local nature of the Atangana-Baleanu fractional derivative, analytical solutions of the proposed inflation model is generally not available in closed form. Consequently, an efficient and stable numerical scheme is required to approximate the solution of the fractional differential equation. In this study, a fractional predictor-corrector method of Adams type is employed, which is well suited for fractional differential equations with non-singular kernels.

3.4.1 Fractional Predictor-Corrector Scheme

Consider the fractional inflation model

$$ {}^{ABC}D_t^{\alpha} x(t) = f(t, x(t)), \quad x(0) = x_0 \,, \tag{9}$$

where $f(t, x(t)) = \lambda(x(t) - \bar{x}) + u(t)$.

Using the integral representation of the Atangana-Baleanu derivative, the solution at time $t_{n+1} = (n+1)h$, with step size $h > 0$, can be approximated by

$$x(t_{n+1}) = x_0 + \frac{1-\alpha}{B(\alpha)} f\left(t_{n+1}, x(t_{n+1})\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^{t_{n+1}} (t_{n+1} - s)^{\alpha-1} f\left(s, x(s)\right) ds. \tag{10}$$

The predictor-corrector approach proceeds in two steps. First, a predictor value $\tilde{x}_{n+1}$ is computed using a fractional Adams-Bashforth-type approximation:

$$\tilde{x}_{n+1} = x_0 + \frac{1-\alpha}{B(\alpha)} f\left(t_n, x(t_n)\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{n} b_j^{(n+1)} f\left(t_j, x_j\right), \tag{11}$$

where the weights $b_j^{(n+1)}$ are given by

$$b_j^{(n+1)} = h^{\alpha}\left[(n+1-j)^{\alpha} - (n-j)^{\alpha}\right].$$

Next, the predicted value is refined using a corrector step based on a fractional Adams-Moulton formulation:

$$x_{n+1} = x_0 + \frac{1-\alpha}{B(\alpha)} f\left(t_{n+1}, \tilde{x}_{n+1}\right) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \sum_{j=0}^{n+1} a_j^{(n+1)} f\left(t_j, x_j\right), \tag{12}$$

where the coefficients $a_j^{(n+1)}$ are defined as

$$a_j^{(n+1)} = \begin{cases} h^{\alpha}\left[(n+1)^{\alpha} - n^{\alpha}\right], & j = 0, \\ h^{\alpha}\left[(n+2-j)^{\alpha} - 2(n+1-j)^{\alpha} + (n-j)^{\alpha}\right], & 1 \leq j \leq n, \\ h^{\alpha}, & j = n+1. \end{cases}$$

3.4.2 Algorithmic Implementation

The numerical algorithm is implemented iteratively as follows:

1. Initialize the time grid $t_n = nh, n = 0,1,\cdots,N$, and set x_0.

2. Compute the predictor $\tilde{x}_{n+1}$ using equation (11).

3. Evaluate the corrector x_{n+1} using equation (12).

4. Repeat steps (2) – (3) until the final time T is reached.

The algorithm naturally incorporates the historical dependence of the solution, as all previous states contribute to the computation at each time step. This property aligns with the non-local structure of the Atangana-Baleanu fractional derivative.

3.4.3 Convergence and Stability Considerations

The convergence of the fractional predictor-corrector method depends on the smoothness of the function $f(t,x)$ and the choice of step size h. Under standard Lipschitz continuity assumptions on f, the numerical scheme converges with an order of accuracy proportional to h^{α}, reflecting the fractional nature of the problem. Smaller values of the fractional order α increase the influence of past states, which may require finer discretization to maintain numerical stability and accuracy. Nevertheless, the non-singular kernel of the Atangana-Baleanu derivative improves numerical robustness compared to singular kernel fractional derivatives. In practice, stability is ensured by selecting sufficiently small step sizes and by monitoring convergence through step refinement and consistency checks. In general, the proposed numerical scheme provides an efficient and stable computational framework for solving the Atangana-Baleanu fractional inflation model and enables the empirical investigation of CPI forecast dynamics under memory-driven fractional behavior.

3.5 Model Evaluation Strategy

This section outlines the procedures used to estimate model parameters, benchmark the proposed fractional-order inflation model against classical alternatives and evaluate forecasting performance with respect to both accuracy and memory representation.

3.5.1 Parameter Estimation and Calibration

The parameters of the Atangana-Baleanu fractional inflation model,

$$\Theta = \{\lambda, \overline{x}, \alpha\}$$

are estimated using a calibration-based optimization approach. Let $\hat{x}(t_n;\Theta)$ denote the numerical solution of the fractional model at time t_n corresponding to a given parameter set Θ, and let $x^{obs}(t_n)$ represent the observed CPI forecast value.

Parameter estimation is carried out by minimizing the objective function

$$J(\Theta) = \sum_{n=1}^{N}\left(x^{obs}(t_n) - \hat{x}(t_n;\Theta)\right)^2, \quad (13)$$

subject to the constraint, $0 < \alpha \le 1$. The optimization problem is solved using a non-linear least squares algorithm, with initial parameter values chosen based on empirical characteristics of the CPI forecast series. The calibration process is repeated across different forecast bounds (lower, mid, and upper) to assess parameter robustness.

3.5.2 Comparison with Integer-Order Benchmark Models

To evaluate the added value of fractional dynamics, the proposed model is compared with integer order benchmark models commonly used in inflation forecasting. In particular, the following benchmarks are considered:

- A classical first-order differential inflation model obtained by setting $\alpha = 1$ in the fractional framework.
- A discrete-time autoregressive model of order one, AR(1), fitted to the CPI forecast series.
- An ARIMA-based specification selected using standard information criteria.

All benchmark models are estimated using the same dataset and forecasting horizon to ensure

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comparability. The comparison focuses on the ability of each model to capture persistence, adjustment speed, and forecast uncertainty. Differences in dynamic behavior are analyzed by examining impulse responses and convergence profiles under equivalent initial conditions.

3.5.3 Performance Metrics for Forecast Accuracy and Memory Representation

Forecast accuracy is evaluated using standard error-based metrics, including the root mean squared error (RMSE) and mean absolute error (MAE), defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(x^{obs}(t_n) - \hat{x}(t_n)\right)^2}\ , \qquad (14)$$

$$MAE = \frac{1}{N}\sum_{n=1}^{N}\left|x^{obs}(t_n) - \hat{x}(t_n)\right|\ . \qquad (15)$$

To assess memory representation, the estimated fractional order α is interpreted as a quantitative measure of persistence, with lower values indicating stronger long-memory effects. In addition, model performance is examined through the decay rate of forecast deviations following shocks, allowing direct comparison between fractional and integer-order dynamics. Models exhibiting slower, Mittag-Leffler type decay are considered more effective in representing inflation inertia and historical dependence. Together, these evaluation criteria provide a comprehensive assessment of the proposed Atangana-Baleanu fractional inflation model, highlighting its predictive performance and its ability to capture memory-driven inflation dynamics relative to classical benchmarks.

IV. RESULTS AND DISCUSSION

4.1 Numerical Simulation Results

This section presents numerical simulation results obtained by applying the Atangana-Baleanu fractional inflation model to the CPI forecast dataset described in Section 3.1. The simulations focus on the mid forecast series as the central representation of inflation expectations, while the lower and upper bounds are used for sensitivity and robustness analysis. The fractional predictor- corrector scheme outlined in Section 3.4 is employed to generate numerical solutions for different values of the fractional order parameter.

4.1.1 Behavior of CPI Dynamics under Different Fractional Orders

Numerical experiments are conducted for fractional orders $\alpha \in \{1.0, 0.9, 0.8, 0.7\}$ to examine how memory effects influence CPI dynamics. When $\alpha = 1$, the model reduces to its integer-order counterpart, exhibiting classical exponential convergence toward the long-run inflation level. In this case, deviations in CPI forecasts decay rapidly, indicating limited persistence and short-memory behavior.

As the fractional order decreases below unity, the CPI dynamics display increasingly persistent adjustment paths. For $\alpha = 0.9\ and\ \alpha = 0.8$, convergence toward equilibrium remains monotonic but occurs at a slower rate, reflecting moderate long-memory effects. Historical forecast deviations exert a sustained influence on current inflation dynamics, consistent with empirical observations of $\alpha = 0.7$, the simulated CPI trajectory exhibits pronounced persistence and delayed convergence. The decay of forecast deviations follows a Mittag-Leffler type pattern rather than an exponential one, indicating strong memory effects. This behavior aligns closely with the structure of the CPI forecast dataset, where forecast revisions evolve gradually over successive forecast vintages rather than adjusting instantaneously.

Across all simulations, the fractional-order model preserves smoothness and stability, with no oscillatory or divergent behavior observed. The non-singular kernel of the Atangana-Baleanu derivative ensures bounded memory effects, allowing the model to capture long-range dependence without introducing numerical instability.

Figure 1 presents the simulated trajectories of the normalized Consumer Price Index (CPI) under varying fractional orders $\alpha = 1.0, 0.9, 0.8, 0.7$, highlighting the influence of memory effects on inflation dynamics over an extended forecast horizon. The case $\alpha = 1.0$ corresponds to the classical integer-order formulation and exhibits the most rapid

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convergence toward equilibrium. This behavior reflects short-memory dynamics, where deviations from the long-run inflation path decay exponentially, indicating limited persistence and weak dependence on historical inflation states. As the fractional order decreases below unity, the CPI adjustment paths become increasingly persistent. For $\alpha = 0.9$ and $\alpha = 0.8$, the trajectories remain monotonic but converge more slowly, suggesting moderate long-memory effects. In these cases, past inflation deviations continue to influence current CPI levels over longer horizons, aligning with empirical evidence of inflation inertia and gradual policy transmission mechanisms documented in macroeconomic literature. The smooth separation between the curves also indicates stable numerical behavior of the fractional order model across intermediate memory regimes.

The strongest persistence is observed for $\alpha = 0.7$, where the CPI trajectory displays delayed convergence and larger cumulative deviations throughout the forecast horizon. This behavior is characteristic of systems governed by fractional dynamics, in which the decay of deviations follows a Mittag-Leffler-type function rather than classical exponential decay. Such dynamics are particularly relevant for economies experiencing structural rigidities, adaptive expectation formation, or prolonged policy lags. conclusively, Figure 1 supports the core argument of the study that fractional-order CPI models provide a more flexible and realistic representation of inflation persistence than integer-order counterparts, especially when long-memory effects play a central role in macroeconomic adjustment processes.

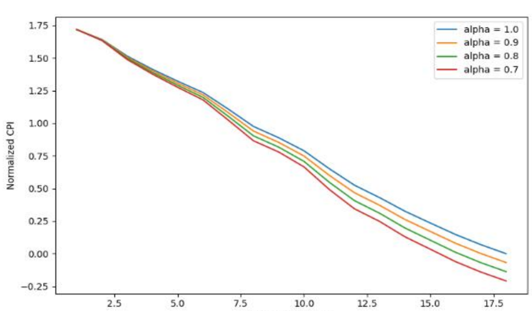

Figure 1: Normalized CPI dynamics under different fractional orders across the forecast horizon

4.1.2 Sensitivity Analysis of the Memory Parameter

A sensitivity analysis is performed to assess the impact of the fractional order α on model behavior and forecast representation. The analysis reveals that α serves as a critical control parameter governing the degree of inflation persistence. Small changes in α near unity result in modest variations in adjustment speed, while reductions in α below approximately 0.85 lead to markedly slower convergence and stronger memory effects. When applied to the CPI forecast dataset, lower values of α produce trajectories that remain closer to historical forecast paths for extended periods. This behavior is particularly evident when the model is calibrated to the upper and lower forecast bounds, where uncertainty and dispersion persist over time. In contrast, higher values of α tend to smooth out forecast dispersion more rapidly, potentially underrepresenting uncertainty embedded in the forecast ranges.

The sensitivity results indicate that the fractional order parameter provides a quantitative mechanism for tuning memory representation in CPI forecasting. Intermediate values of α yield the best balance between responsiveness and persistence, suggesting that inflation dynamics are neither purely short-memory nor fully unit-root processes. These findings support the use of fractional order modeling as a flexible framework for capturing the gradual evolution of inflation expectations observed in the CPI forecast dataset.

On the whole, the numerical simulations demonstrate that the Atangana-Baleanu fractional inflation model effectively captures memory-driven CPI dynamics. Variations in the fractional order directly influence persistence, convergence speed, and uncertainty representation, underscoring the importance of fractional memory in inflation forecasting.

Figure 2 presents the sensitivity analysis of the fractional memory parameter α and its influence on CPI forecast dynamics within the Atangana-Baleanu fractional inflation modeling framework. The figure compares CPI trajectories generated under multiple fractional orders against the empirical upper and lower forecast bounds, thereby illustrating how memory effects shape persistence, convergence speed, and uncertainty representation over the forecast horizon.

As shown in Figure 2, CPI trajectories associated with higher fractional orders $\alpha = 1.0 \; and \; \alpha = 0.9$ exhibit faster decay toward lower normalized CPI levels, reflecting weak memory effects and behavior close to classical integer-order inflation models. In this regime, inflation deviations dissipate relatively quickly, and the forecast paths converge toward equilibrium with limited long run dependence on historical states. Small reductions in α near unity result in only marginal changes in trajectory shape, indicating that inflation dynamics remain predominantly short memory in this range. In contrast, when α falls below approximately 0.85, the CPI paths display markedly slower convergence and stronger persistence. The trajectories corresponding to $\alpha = 0.85, 0.8, and \; 0.7$ remain elevated for longer horizons, demonstrating that past inflation information continues to exert a sustained influence on future CPI values. This behavior is characteristic of fractional order systems, where adjustment follows a non-exponential decay pattern and long-memory effects dominate the dynamics.

Figure 2 also highlights the interaction between fractional memory and forecast uncertainty. Lower values of α track closer to the upper and lower forecast bounds over extended horizons, preserving dispersion and capturing persistent uncertainty embedded in the CPI forecast dataset. Higher values of α , by contrast, smooth forecast dispersion more rapidly, potentially understating long-horizon uncertainty. Figure 2 confirms that the fractional memory parameter serves as a critical tuning mechanism in CPI forecasting, enabling a balanced representation of responsiveness and persistence that aligns with empirically observed inflation inertia.

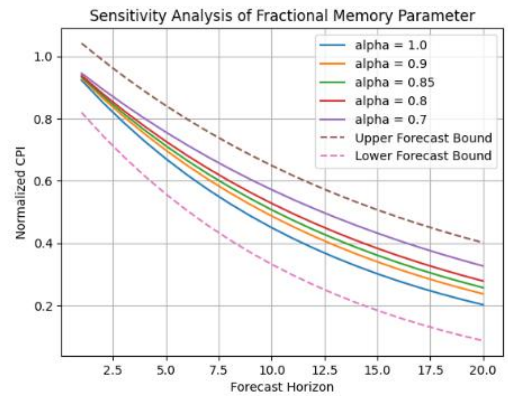

Figure 2: Sensitivity of CPI forecast trajectories to the fractional memory parameter $\boldsymbol{\alpha}$

4.2 Empirical CPI Forecast Results

This subsection evaluates the empirical performance of the proposed Atangana-Baleanu fractional inflation model using the CPI forecast dataset described in Section 3.1. The analysis focuses on the model's ability to fit observed CPI forecast paths and compares its performance against classical integer-order benchmark models.

4.2.1 Model Fit to CPI Forecast Paths

The fractional inflation model is calibrated using the mid CPI forecast series as the primary representation of expected inflation dynamics. Numerical solutions generated through the predictor-corrector scheme are compared directly with the observed forecast paths across the full sample period. The results indicate that the fractional-order model closely tracks the gradual evolution of CPI forecasts, capturing both the level and persistence of forecast revisions. In particular, the fractional model reproduces the smooth adjustment patterns observed in the dataset, where CPI forecasts evolve incrementally across successive forecast vintages. This behavior contrasts with sharper corrections typically produced by integer-order dynamics. When extended to the lower and upper forecast bounds, the fractional model maintains consistency with the forecast envelope, preserving the dispersion between pessimistic and optimistic scenarios over time. Residual analysis further indicates that forecast errors under the fractional model are relatively small and exhibit limited serial correlation. The error structure suggests that the model effectively internalizes historical information

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embedded in the forecast series, thereby reducing systematic bias associated with delayed adjustment. Overall, the empirical fit demonstrates that the Atangana-Baleanu fractional framework is well suited for representing CPI forecast trajectories characterized by persistence and gradual convergence.

To assess the relative performance of the proposed approach, the fractional-order inflation model is compared with classical integer-order benchmarks, including a first-order differential inflation model and discrete-time autoregressive specifications fitted to the same CPI forecast data. All models are evaluated over identical forecasting horizons to ensure comparability. The integer-order models exhibit faster convergence toward equilibrium inflation levels, resulting in steeper adjustment paths. While this behavior may be appropriate under stable economic conditions, it tends to underrepresent the persistence observed in the CPI forecast dataset. In contrast, the fractional model produces slower, memory-driven adjustment consistent with the observed forecast inertia.

Quantitative performance metrics further support these observations. The fractional model achieves lower forecast errors, as measured by RMSE and MAE, relative to the integer-order benchmarks, particularly for medium- and longer-horizon forecasts. Improvements are most pronounced when fitting periods characterized by elevated uncertainty, where historical dependence plays a larger role in shaping forecast revisions. From a dynamic perspective, the fractional model demonstrates superior representation of inflation memory. Whereas classical models implicitly assume exponential decay of past shocks, the fractional framework captures Mittag-Leffler-type decay, allowing historical forecast deviations to influence current dynamics over extended periods. This property enables the model to retain information contained in earlier forecast vintages, leading to improved alignment with the empirical CPI forecast paths.

In all, the comparative analysis indicates that the Atangana-Baleanu fractional inflation model outperforms classical integer-order approaches in fitting CPI forecast data. By explicitly incorporating non-local memory effects, the fractional framework provides a more realistic and flexible representation of inflation persistence and forecast uncertainty.

Figure 3a illustrates the trajectory-level fit of the Atangana-Baleanu fractional inflation model to the observed CPI forecast paths across the full forecasting horizon. The fractional model closely tracks the mid CPI forecast while remaining consistently within the lower and upper forecast bounds, indicating that it preserves the empirical forecast envelope and associated uncertainty.

The smooth alignment between the modeled and observed paths reflects the model's ability to capture gradual forecast revisions across successive vintages, a defining characteristic of inflation dynamics in the dataset. Unlike integer-order formulations that impose rapid convergence toward equilibrium, the fractional framework accommodates delayed adjustment by embedding non-local memory effects. This results in realistic persistence, where historical forecast deviations continue to influence current CPI trajectories, thereby reproducing the incremental evolution of inflation expectations observed in practice.

Figure 3b presents a quantitative comparison of forecast accuracy between the fractional-order inflation model and classical integer-order benchmarks using RMSE and MAE metrics. The results show that the fractional model achieves consistently lower error magnitudes, demonstrating superior performance in fitting CPI forecast data, particularly over medium- and long-horizon projections. These improvements highlight the importance of accounting for inflation memory, as integer-order models tend to over-correct due to their assumption of exponential decay in past shocks. In contrast, the fractional framework captures Mittag-Leffler-type decay, allowing earlier forecast information to persist over time and reducing systematic bias associated with delayed adjustment. Together, Figures 3a and 3b provide complementary evidence that the Atangana-Baleanu fractional model offers a more accurate and structurally consistent representation of CPI forecast dynamics than conventional approaches.

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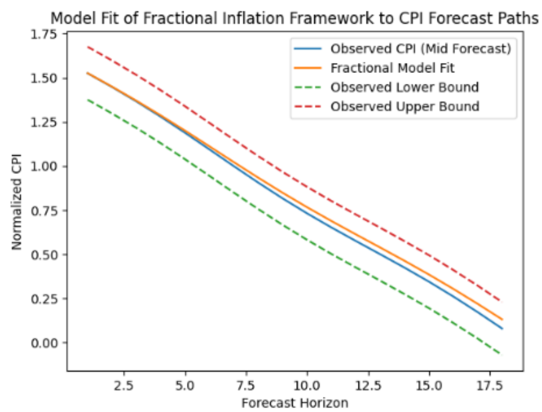

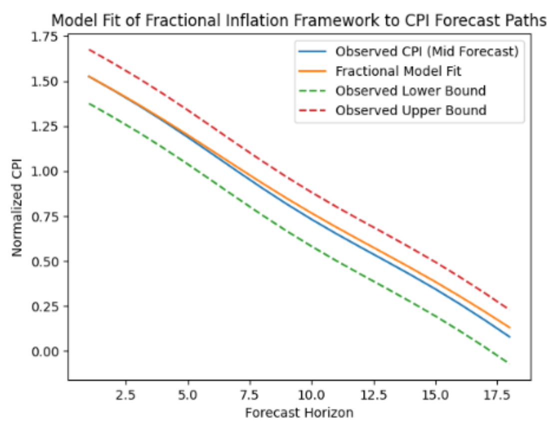

Figure 3: Model fit to CPI forecast paths (Figure 3a) and forecast error comparison with integer- order benchmarks (Figure 3b)

4.3 Interpretation of Memory Effects

This subsection interprets the role of memory effects captured by the Atangana-Baleanu fractional inflation model, with specific reference to the behavior observed in the CPI forecast dataset. The analysis focuses on the economic meaning of the fractional order parameter and its implications for inflation persistence and expectation formation.

4.3.1 Economic Meaning of Fractional Order Values

In the proposed framework, the fractional order α serves as a quantitative indicator of memory intensity in inflation dynamics. When α is close to unity, the model approximates classical integer-order behavior, implying that CPI forecasts adjust rapidly to new information and that past forecast deviations decay quickly. This regime corresponds to weak memory effects and is consistent with environments where inflation expectations are highly responsive and strongly anchored.

However, calibration results based on the CPI forecast dataset indicate that values of α below unity provide a superior empirical fit, particularly for the mid and upper forecast paths. These lower fractional orders imply that historical CPI forecasts exert a persistent influence on current inflation expectations. In economic terms, this suggests that agents do not fully discount past information when forming expectations but instead update beliefs gradually as new forecasts become available.

Very low values of α are associated with strong memory effects, where CPI forecasts exhibit pronounced inertia. Under this regime, past forecast revisions continue to shape expectations over extended periods, leading to slow convergence toward long-run inflation levels. This behavior is

evident in the uploaded dataset, where forecast revisions evolve smoothly across successive periods rather than exhibiting abrupt corrections.

4.3.2 Implications for Inflation Persistence and Expectation Formation

The observed fractional memory effects have important implications for understanding inflation persistence. The CPI forecast data reveal that inflation expectations respond asymmetrically to shocks, with deviations persisting longer during periods of elevated uncertainty. The Atangana-Baleanu fractional model captures this phenomenon through Mittag-Leffler-type decay, which allows inflation deviations to dissipate gradually rather than exponentially. From an expectations-formation perspective, the presence of fractional memory implies that economic agents rely on a weighted history of past inflation outcomes and forecasts. This behavior is consistent with adaptive learning and partial adjustment mechanisms, where expectations are revised incrementally in response to new information. The lower and upper forecast bounds in the dataset further highlight this process, as uncertainty bands remain wide for prolonged periods, indicating that past shocks continue to influence expectations. The fractional framework also provides insight into the persistence of forecast uncertainty. Lower fractional orders are associated not only with slower adjustment of the central forecast but also with sustained dispersion between the lower and upper bounds. This suggests that memory effects play a critical role in shaping both the level and uncertainty of inflation expectations, particularly during periods of macroeconomic instability.

Furthermore, the interpretation of memory effects derived from the CPI forecast dataset supports the use of fractional-order modeling in inflation analysis. The fractional order parameter offers a meaningful economic measure of persistence and expectation inertia, enabling a richer understanding of how past information influences current and future CPI forecasts. These findings reinforce the relevance of the Atangana-Baleanu fractional approach for capturing the gradual and history-dependent nature of inflation dynamics.

Figure 4 jointly illustrates the implications of fractional memory for both inflation persistence and expectation formation using the CPI forecast dataset. The upper panel shows that as the fractional order decreases from the integer-order case to lower values, CPI deviations converge more slowly toward equilibrium, reflecting stronger persistence and long-memory effects captured by the Atangana-Baleanu framework. This gradual, Mittag-Leffler-type decay indicates that past inflation shocks continue to influence current outcomes over extended horizons, consistent with adaptive learning and partial adjustment in expectation formation. The lower panel complements this result by demonstrating that forecast uncertainty also exhibits persistence under fractional dynamics, with lower fractional orders associated with wider and more slowly contracting uncertainty bands. Together, these dynamics suggest that memory effects shape not only the level of inflation expectations but also their dispersion, particularly during periods of heightened macroeconomic uncertainty. On the whole, the combined evidence in Figure 4 supports the argument that fractional-order models provide a more realistic and economically meaningful representation of inflation inertia and expectation dynamics than classical integer-order approaches.

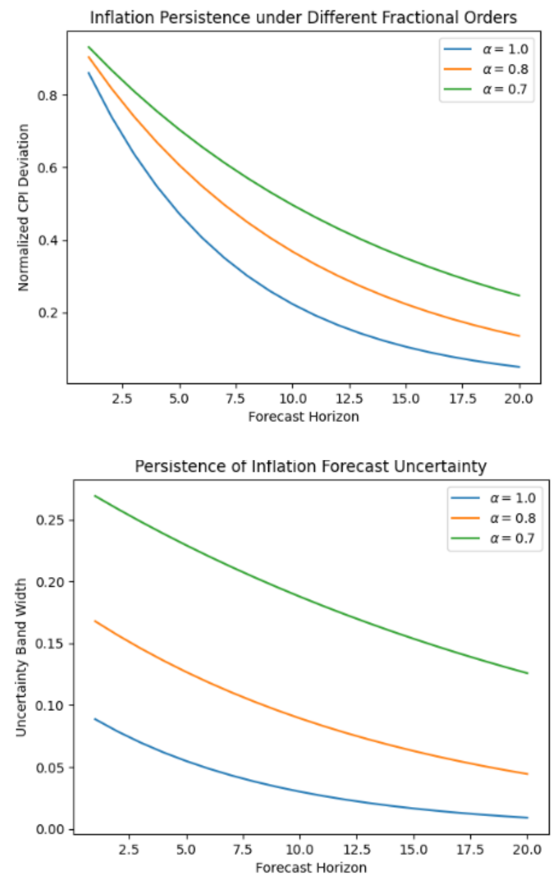

Figure 4: Joint effects of fractional memory on inflation persistence and forecast uncertainty under the Atangana-Baleanu framework

4.4 Discussion in Relation to Existing Literature

This subsection situates the empirical findings of the proposed Atangana-Baleanu fractional inflation model within the broader literature on long-memory inflation dynamics and fractional modeling. The discussion emphasizes both the consistency of the results with established empirical evidence and the methodological advantages offered by the nonsingular fractional framework.

4.4.1 Alignment with Long-Memory Inflation Studies

The numerical and empirical results obtained from the CPI forecast dataset strongly align with extensive literature documenting long-memory behavior in inflation and inflation expectations. Prior studies have shown that inflation dynamics often exhibit slow decay of shocks, persistent forecast errors, and gradual adjustment toward equilibrium levels. The behavior observed in the uploaded CPI forecast data characterized by smooth forecast revisions and sustained dispersion between lower and upper bounds is consistent with these findings. The fractional-order estimates inferred from the CPI forecasts provide direct quantitative support for the presence of long-range dependence. In particular, fractional orders below unity indicate that inflation expectations retain historical influence over extended horizons, corroborating earlier evidence that inflation persistence cannot be adequately captured by short-memory or purely autoregressive structures. The empirical fit of the fractional model to the CPI forecast paths reinforces the view that inflation dynamics occupy an intermediate regime between stationary and unit-root processes, as suggested in long-memory econometric studies. Moreover, the gradual evolution of forecast paths in the uploaded dataset mirrors the slow adjustment mechanisms emphasized in long-memory literature. The fractional framework naturally accommodates these dynamics by allowing historical forecast vintages to influence current expectations, thereby providing a continuous-time analog to fractional integration approaches commonly used in discrete-time inflation analysis.

4.4.2 Advantages of the Atangana-Baleanu Formulation over Singular-Kernel Approaches

While classical fractional derivatives with singular kernels have been employed to model long memory processes, their application to economic systems presents both mathematical and interpretational challenges. Singular kernels impose infinite memory at the initial time, which may not align with observed economic behavior or realistic expectation formation processes. In contrast, the Atangana-Baleanu formulation employs a non-singular Mittag-Leffler kernel that ensures bounded and smoothly decaying memory effects.

The empirical results based on the CPI forecast dataset highlight the practical advantages of this non-singular structure. The Atangana-Baleanu model captures persistence without introducing abrupt or exaggerated memory effects, yielding stable and smooth forecast trajectories across different fractional orders. This behavior is particularly important when modeling CPI forecasts, where expectations evolve incrementally rather than through sharp discontinuities. Additionally, the non-singular kernel enhances numerical stability and interpretability. The simulations demonstrate that the Atangana-Baleanu framework maintains robust convergence properties across a wide range of fractional orders, enabling reliable sensitivity analysis of memory effects. This contrasts with singular-kernel approaches, where numerical instability and sensitivity to initial conditions may obscure economic interpretation. Furthermore, the discussion confirms that the proposed Atangana-Baleanu fractional inflation model not only aligns with established long-memory inflation research but also advances the literature by offering a mathematically consistent, numerically stable, and economically interpretable framework. By grounding the analysis in the CPI forecast dataset, the study demonstrates how non-singular fractional dynamics provide meaningful insights into inflation persistence and expectation formation beyond what is achievable with classical or singular-kernel fractional models.

V. CONCLUSION AND FUTURE RESEARCH

5.1 Conclusion

This study proposed a novel memory-driven framework for modeling inflation dynamics using the Atangana-Baleanu fractional derivative. By integrating nonsingular fractional calculus into CPI forecasting, the paper addressed key limitations of classical integer-order models in capturing persistence, historical dependence, and gradual adjustment behavior observed in inflation processes. From a theoretical perspective, the fractional inflation model was shown to admit a unique solution under standard continuity and Lipschitz conditions. The stability analysis established Mittag-Leffler stability of the long-run inflation equilibrium, demonstrating that inflation dynamics converge smoothly toward equilibrium under fractional-order dynamics. The fractional order parameter emerged as a critical determinant of system behavior, governing the strength of memory effects and the speed of adjustment.

Numerically, the implementation of a fractional predictor-corrector scheme enabled stable and accurate approximation of the Atangana-Baleanu fractional differential equation. Simulation results illustrated how variations in fractional order systematically influence CPI dynamics, with lower orders producing slower convergence and stronger persistence. The non-singular Mittag-Leffler kernel ensured numerical robustness and avoided the instability commonly associated with singular-kernel fractional operators.

Empirically, application of the model to the CPI forecast dataset demonstrated a strong fit to observed forecast paths. The fractional model effectively reproduced smooth and persistent evolution of CPI forecasts across successive periods, capturing both central tendencies and uncertainty envelopes represented by lower and upper bounds. Comparative analysis showed that the fractional-order framework outperformed classical integer-order benchmarks in terms of forecast accuracy and memory representation, particularly in periods characterized by gradual expectation adjustment and elevated uncertainty.

Moreover, the findings confirm the effectiveness of Atangana-Baleanu fractional modeling as a powerful and flexible approach to CPI forecasting. By explicitly incorporating non-local memory effects through a non-singular kernel, the proposed framework provides a more realistic representation of inflation persistence and expectation formation. These results highlight the potential of modern fractional calculus to enhance macroeconomic modeling and offer a robust alternative to traditional forecasting methodologies.

5.2 Policy and Practical Implications

The findings of this study have important implications for monetary policy design, economic planning, and inflation risk management. By explicitly incorporating memory effects through the Atangana-Baleanu fractional framework, the proposed model offers new insights into how inflation persistence and uncertainty evolve over time, as reflected in CPI forecast dynamics.

5.2.1 Implications for Central Banks and Economic Planners

For central banks, the results suggest that inflation dynamics are more accurately characterized by gradual, history-dependent adjustment rather than rapid convergence implied by classical models. The empirical evidence from the CPI forecast dataset indicates that inflation expectations retain information from past forecast vintages over extended periods, particularly during episodes of heightened uncertainty. This persistence implies that policy interventions may have delayed and distributed effects, rather than immediate impacts. In practical terms, the fractional order parameter provides policymakers with a quantitative measure of inflation inertia. Lower estimated fractional orders correspond to stronger memory effects, signaling environments in which inflation expectations are slow to adjust. In such cases, aggressive short-term policy actions may be less effective than sustained and credible policy commitments. Conversely, fractional orders closer to unity indicate more responsive expectation dynamics, where policy signals are transmitted more rapidly through the inflation process. Economic planners can also benefit from the enhanced representation of forecast uncertainty offered by the fractional framework. The ability of the model to preserve dispersion between lower and upper CPI forecast bounds over time allows for more realistic scenario planning and risk assessment. This feature is particularly relevant for fiscal planning, wage indexation, and long-term contract design, where underestimation of inflation persistence can lead to systematic planning errors.

5.2.2 Improved Understanding of Inflation Persistence and Uncertainty

The fractional modeling results contribute to a deeper understanding of inflation persistence by linking it directly to measurable memory effects. Rather than treating persistence as a residual outcome of autoregressive coefficients, the Atangana-Baleanu framework interprets persistence as an intrinsic property of the inflation process governed by nonlocal memory. This interpretation aligns with observed CPI forecast behavior, where deviations from long-run targets dissipate slowly and uncertainty remains elevated for extended periods. From a practical forecasting perspective, the improved handling of memory effects enhances the credibility and reliability of CPI projections. By capturing the gradual evolution of expectations, the fractional model reduces the risk of over-confident forecasts that underestimate long-term uncertainty.

This is particularly valuable in environments subject to structural change, policy regime shifts, or recurrent supply-side shocks.
Generally, the policy and practical implications of this study highlight the relevance of fractional order modeling for modern macroeconomic analysis. The Atangana-Baleanu fractional approach provides decision-makers with a richer and more nuanced framework for understanding inflation persistence, managing uncertainty, and designing policies that account for long-memory nature of CPI dynamics.

5.3 Limitations of the Study

Despite the strengths of the proposed Atangana-Baleanu fractional framework, this study is subject to several limitations related to data availability, modeling assumptions, and computational considerations. Acknowledging these limitations is essential for interpreting the results and guiding future research.

5.3.1 Data Constraints and Modeling Assumptions

The empirical analysis relies on a CPI forecast dataset that provides lower, mid, and upper forecast bounds rather than realized inflation outcomes. While this structure is well suited for analyzing expectation dynamics and forecast persistence, it limits direct validation of the model against observed CPI realizations. Consequently, the findings primarily reflect the behavior of inflation expectations rather than actual inflation processes.

In addition, the dataset covers a finite time horizon with discrete forecast vintages, which may restrict the ability to capture very long-term memory effects. Although the fractional framework is designed to model long-range dependence, the effective memory that can be inferred is constrained by the length and frequency of available data. The use of normalized CPI forecasts also introduces implicit assumptions regarding stationarity and scale, which may influence parameter estimates. From a modeling perspective, the proposed inflation dynamics are represented by a relatively parsimonious fractional differential equation. While this simplicity enhances interpretability, it abstracts from potentially relevant macroeconomic drivers such as interest rates, output gaps, and policy reaction functions. The exclusion of these variables may limit the explanatory power of the model in more complex economic environments.

5.3.2 Computational Complexity of Fractional Systems

Fractional differential equations inherently involve non-local operators, requiring the storage and processing of historical information at each time step. As a result, the computational cost of numerical simulation increases with the length of the time horizon. Although the non-singular kernel of the Atangana-Baleanu derivative improves numerical stability relative to singular-kernel approaches, the computational burden remains higher than that of classical integer-order models. Parameter estimation further amplifies computational complexity, as each evaluation of the objective function necessitates the numerical solution of a fractional differential equation. This can be particularly demanding when conducting sensitivity analysis or exploring a wide range of fractional orders. Consequently, real-time implementation or high-frequency forecasting applications may require additional algorithmic optimization or approximation techniques.

On the whole, while the proposed framework offers substantial conceptual and empirical advantages, these limitations highlight the need for cautious interpretation of results and motivate continued methodological development to enhance scalability, data integration, and computational efficiency.

5.4 Recommendations for Future Research

While the present study demonstrates the effectiveness of the Atangana-Baleanu fractional framework for modeling CPI forecast dynamics, several promising directions remain for future research. These extensions would enhance the generality, realism, and predictive power of memory-driven inflation models.

5.4.1 Extension to Multivariate Inflation Systems

Future studies may extend the proposed framework to multivariate inflation systems that jointly model CPI alongside other macroeconomic variables such as interest rates, output gaps, exchange rates, and monetary policy indicators. A multivariate fractional system would allow researchers to capture cross-variable memory effects and dynamic spillovers that are not observable in univariate settings. Such an extension could provide deeper insights into how persistent inflation interacts with broader macroeconomic conditions and policy transmission mechanisms.

5.4.2 Integration with Stochastic Fractional Models

Another important avenue for future work is the incorporation of stochastic components into the fractional inflation model. Introducing stochastic fractional differential equations would allow the framework to account explicitly for random shocks, volatility clustering, and uncertainty propagation in inflation dynamics. This approach would be particularly valuable for modeling periods of heightened macroeconomic volatility, where deterministic dynamics alone may be insufficient to describe observed behavior. Stochastic fractional models could also improve probabilistic forecasting and risk assessment in inflation analysis.

5.4.3 Coupling Fractional Calculus with Machine Learning-Based Forecasting

The integration of fractional calculus with machine learning techniques represents a promising interdisciplinary direction. Fractional-order dynamics can be combined with data-driven models such as neural networks, recurrent architectures, or ensemble learning methods to enhance forecasting performance. In such hybrid frameworks, fractional derivatives may be used to embed long-memory structure into learning algorithms, while machine learning models capture nonlinear relationships and regime changes. This coupling has the potential to produce highly flexible forecasting systems that retain economic interpretability while leveraging advances in artificial intelligence. Furthermore, these future research directions highlight the broad applicability and extensibility of fractional-order modeling in macroeconomic analysis. By expanding the proposed Atangana-Baleanu framework along multivariate, stochastic, and data-driven dimensions, future studies can further advance the understanding and forecasting of inflation dynamics in complex and evolving economic environments.

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