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Comparative Analysis of Singular Value Decomposition and Cholesky Decomposition Methods for Solving Large Scale Linear Systems

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Abstract- This study presents a comparative analysis of matrix decomposition methods for solving large-scale linear systems, focusing on computational efficiency, numerical stability, and applicability across different problem domains. Matrix decomposition is fundamental in numerical linear algebra, as it simplifies complex systems into forms that are easier to solve. The study reviews classical methods of SVD and Cholesky decomposition, highlighting their strengths and limitations. SVD extend decomposition to spectral analysis and dimensionality reduction, making them valuable in machine learning and data science. Cholesky decomposition, in contrast, offers speed and stability in positive-definite systems. Applications in optimization, artificial intelligence, recommender systems and structural engineering are examined to demonstrate practical relevance. The findings show that method performance is problem-dependent,with trade-offs between speed, accuracy, and scalability. The study concludes that although classical methods remain foundational, modern large-scale and sparse problems require advances such as randomized and hybrid decomposition techniques

I. INTRODUCTION

Matrix decomposition, also known as matrix factorization, is a fundamental tool in linear algebra and numerical analysis that simplifies complex matrix operations by breaking them into the product of simpler, structured matrices. Historically, the motivation for decomposition arose from the need to solve large systems of linear equations, which form the backbone of scientific computation, engineering models, and data-driven research. Early techniques such as Gaussian elimination laid the foundation for systematic approaches to matrix reduction, but it was not until the mid-20th century with the advent of digital computers that decomposition methods gained prominence (Trefethen, 1997). Householder's Principles of Numerical Analysis emphasized the role of partitioned matrices and block-based LU decomposition as an efficient strategy for high speed computation, marking a significant departure from purely manual methods (Householder 1954, Lu, 2021 & Chandra, 2007). These advancements were not only theoretical but also practical, enabling large-scale problems in geology, physics, and engineering to be solved more reliably.

In contemporary applications, decomposition methods are essential for tackling large-scale linear systems that appear in diverse domains. For instance, single value decomposition and cholesky decomposition are widely employed in solving systems of equations, least squares problems, eigenvalue computations, and optimization challenges (Higham, 2002 & Ejikeme 2015). In computer science and machine learning, matrix decomposition plays a critical role in the development of algorithms for dimensionality reduction, collaborative filtering, and feature extraction (Meyer 2000). For example, SVD is foundational in Principal Component Analysis (PCA), which is used for pattern recognition and data compression. Similarly, factorization techniques underpin neural network training via back propagation, reinforcing the link between numerical linear algebra and artificial intelligence (Lu, 2021; Duan, Jiang, and Jain, 2022).

Despite their ubiquity, decomposition methods encounter limitations when applied to modern large-scale datasets and computational systems. Sparse and structured matrices, common in graph theory and scientific computing, pose unique challenges (Trefethen, 2014). Similarly, the Cholesky decomposition is celebrated for its efficiency on symmetric positive-definite matrices, but its stability

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depends heavily on the condition number of the input matrix. Such challenges emphasize the importance of careful selection and adaptation of decomposition techniques depending on problem structure and computational constraints (Demnel 2019) .

Beyond computational science, matrix decomposition has expanded into interdisciplinary applications. In recommender systems, for instance, matrix factorization is employed to address the “sparsity problem” in user-item rating datasets. By integrating review-based collaborative filtering with matrix factorization, Duan et al. (2022) demonstrated that latent factor models can enhance predictive accuracy in cases where data is incomplete. This reflects the versatility of decomposition approaches, as methods originally developed for solving linear systems in mathematics now power modern innovations in artificial intelligence, business analytics, and decision support systems. Such applications reaffirm that matrix decomposition is not merely a mathematical tool but a foundational bridge between theory and practice (Benzi 2002 and Demnel 2007).

Matrix decomposition methods, although powerful, face significant challenges when applied to large-scale linear systems (Golub 2009 and Peng 2022). Classical approaches like Cholesky decomposition was originally developed for smaller systems and do not always scale efficiency. As matrix size increases or when matrices are sparse, these methods often demand substantial memory and computational time due to phenomena such as fill-in, where originally sparse matrices become denser during factorization (Saad 2003 and Gentle 2017). This greatly increase both computational cost and storage requirements, making classical decomposition methods inefficient for high-dimensional problems commonly encountered in data science, engineering, and scientific computing (Lu, 2021). these limitation highlight a gap between the theoretical efficiency of decomposition methods and their practical performance in real-world applications.

The aim of the study is to analyze and compare selected matrix decomposition methods in order to evaluate their effectiveness and efficiency in solving large-scale linear systems. Specifically, the study seeks to provide clear definitions and mathematical proofs of the decomposition methods, examine their numerical stability and accuracy, analyze the trade-offs between computational speed and stability, and compare the methods to determine their suitability for different types of linear systems.

This study is restricted to two matrix decomposition methods: Cholesky decomposition and SV decomposition. Rather than providing detailed derivations of these methods, the study focuses on comparing their computational efficiency and complexity in solving large-scale linear systems, as well as their performance on real-world datasets and randomly generated matrices. Consequently, the scope of this research does not extend to implementations in specialized software packages such as LAPACK, ARPACK, or Suite-sparse.

II. THEORY OF METHODS

2.1 Singular Value Decomposition

In linear algebra, few tools are as powerful and versatile as the Singular Value Decomposition (SVD). While *LU* decomposition targets square matrices for solving linear systems, SVD extends its utility far beyond, encompassing even non-square matrices, whether tall or wide (Ryan 2008). From numerical stability to dimensionality reduction, SVD is a cornerstone in applied mathematics, machine learning, signal processing, and statistics.

Given any $m \times n$ real matrix A, SVD enables us to express A as the product of three matrices:

$$A = U\Sigma V^{T}$$

Here, U is an $m \times m$ orthogonal matrix, Σ is an $m \times n$ diagonal matrix with non-negative real numbers (called singular values) on the diagonal, and V is an $n \times n$ orthogonal matrix.

SVD is more than just a factorization; it unveils the geometry of linear transformations. The columns of V represent the directions in the input space, the diagonal entries of Σ scale them, and the columns of U reorient the result. These insights are critical when analyzing or compressing data, computing pseudoinverses, or solving ill-posed problems.

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Definition 1. (Singular Value Decomposition). Let $A \epsilon R^{m\times n}$. Then there exist matrices $U \epsilon R^{m\times m}$, $V \epsilon R^{n\times n}$, and $\Sigma \epsilon R^{m\times n}$ such that:

$A = U\Sigma V^T$,

where:

- $U^T U = I_m$ (orthogonal),
- $V^T V = I_n$ (orthogonal),
- Σ is diagonal with non-negative real numbers $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$, called singular

values.

Here, r = rank(A) is the number of non-zero singular values.

Theorem 1. (Existence of SVD). Every real matrix $A \epsilon R^{m\times n}$ admits a singular value decomposition.

Proof. The proof is rooted in spectral theory. Consider the symmetric matrices $A^TA \epsilon R^{n\times n}$ and $AA^T \epsilon R^{m\times m}$. Both are positive semi-definite and hence have orthonormal eigenvectors with non-negative eigenvalues. Let:

$A^TA = V \Lambda V^T$,

where Λ is diagonal with eigenvalues $\lambda_i \geq 0$, and V is orthogonal. Define singular values as

$\sigma_i = \sqrt{\lambda_i}$. Set:

$\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_r, 0, \ldots)$, $U = AV\Sigma^\dagger$,

where $\Sigma^\dagger$ is the pseudoinverse of Σ. It can be shown that U is orthogonal, and thus:

$A = U\Sigma V^T$

Remark:

(i). The singular values of A are the square roots of the eigenvalues of ATA.

(ii). The columns of V are the eigenvectors of ATA; those of U are the eigenvectors of AAT.

(iii). The number of non-zero singular values equals the rank of A.

2.1.1 Applications of Singular Value Decomposition Method:

1. Dimensionality reduction: Principal Component Analysis (PCA) is based on SVD.
2. Solving least squares problems: For over determined systems $Ax = b$, SVD yields the minimum norm solution.
3. Data compression: Low-rank approximations use the largest singular values to represent the matrix efficiently.
4. Image processing: SVD compresses grayscale or RGB images by truncating smaller singular values

2.1.2 Singular Value Decomposition: Step-by-Step Method

Let A be an $m \times n$ real matrix.

1. Compute A^TA and find its eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$.
2. The singular values are $\sigma_i = \sqrt{\lambda_i}$.
3. Find the eigenvectors $v_1, \ldots, v_n$ of A^TA. Form $V = [v_1, \ldots, v_n]$.
4. Compute $u_i = \frac{1}{\sigma_i} A v_i$ for $\sigma_i \neq 0$, and form $U = [u_1, \ldots, u_m]$.
5. Form the diagonal matrix Σ with $\sigma_1, \ldots, \sigma_r$ on the diagonal.
6. Then, $A = U\Sigma V^T$

Example 1:

Let

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Step 1: Compute $\boldsymbol{A^TA}$

$$A^TA = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$

Step 2: Compute eigen value of $\boldsymbol{A^TA}$

$$\lambda^2 - 20\lambda + 64 = 0 \quad \Longrightarrow \lambda_1 = 16, \quad \lambda_2 = 4.$$

So, singular values are: $\sigma_1 = \sqrt{16} = 4$ $\sigma_2 = \sqrt{4} = 2$

Step 3: Compute V

Eigenvectors of A^TA are:

$$v_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad v_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Step 4: Compute U

$$u_1 = \frac{1}{4}Av_1 = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{1}{2}Av_2 = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

So:

$$U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}, \quad V^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

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Example 2: Tall Matrix

Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Then

$$A^T A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \sigma_1 = \sigma_2 = 1$$

$$V = I_2, \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So,

$$A = U\Sigma V^T$$

Example 3: Low-Rank Matrix

Let

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$$

Then

$$A^T A = \begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$$

Eigenvalues: $\lambda_1 = 25$, $\lambda_2 = 0$, so $\sigma_1 = 5$, $\sigma_2 = 0$.

Eigenvectors of λ_1 : $v_1 = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$u_1 = \frac{1}{5} A v_1 = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

So,

$$U = \begin{bmatrix} \frac{1}{\sqrt{1.25}} & * \\ \frac{0.5}{\sqrt{1.25}} & * \end{bmatrix} \quad \Sigma = \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} \frac{1}{\sqrt{5}} & * \\ \frac{2}{\sqrt{5}} & * \end{bmatrix}$$

$$A = U\Sigma V^T$$

SVD is a universal matrix decomposition method applicable to any real matrix. It reveals the intrinsic structure of the matrix, identifies its rank, and provides the optimal low-rank approximation. Its numerical stability and theoretical depth make it indispensable in applied linear algebra.

2.2. Cholesky Decomposition:

Solving systems of linear equations efficiently and with numerical stability is of utmost importance. For symmetric and positive definite matrices, a specialized and more efficient decomposition exists—called the Cholesky Decomposition.

Given a symmetric, positive definite matrix $A \in R^{n\times n}$, Cholesky decomposition factors A into the product of a lower triangular matrix and its transpose:

$$A = LL^T$$

where L is a lower triangular matrix with positive diagonal entries, and L^T is its transpose.

Compared to LU decomposition, Cholesky decomposition is approximately twice as efficient and numerically more stable, making it the method of choice in many applications, such as Kalman filters, Monte Carlo simulations, and optimization algorithms, particularly in least squares and machine learning (Golub and Lipson 2013).

Definition 2. (Cholesky Decomposition). Let $A \in R^{n\times n}$ be a symmetric and positive definite matrix. Then there exists a unique lower triangular matrix L with strictly positive diagonal entries such that:

$$A = LL^T.$$

2.2.1 Step-by-Step Method for Cholesky Decomposition

Let $A = [a_{ij}]$ be a symmetric, positive definite matrix. The goal is to find the lower triangular matrix $L = l_{ij}$ such that $A = LL^T$.

Step 1: Initialization

Let L be a $n \times n$ zero matrix.

Step 2: Recursive computation of entries

Compute each entry of L using the following formular:

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} i_{ik}^2}, \text{ for } i = 1,2,3,\dots n.$$

$$l_{ij} = \frac{1}{l_{jj}}\left(a_{ij} - \sum_{k=1}^{j-i} l_{ik}\, l_{jk}\right), \qquad i > j.$$

Step 3: Assemble L and verify

After computing all entries, assemble matrix L and verify.

$$A = LL^T.$$

Example 1:

Let:

$$A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$$

Step 1: Verify Symmetry and Positive Definiteness
The matrix A is symmetric. A quick check shows it is also positive definite (all leading principal minors are positive).
Step 2: Compute L.
Using the cholesky algorithm:

$$l_{11} = \sqrt{4} = 2$$

$$l_{21} = \frac{12}{2} = 6, \quad l_{31} = \frac{-16}{2} = -8$$

$$l_{22} = \sqrt{37 - 6^2} = \sqrt{1} = 1$$

$$l_{32} = \frac{-43 - (-8)(6)}{1} = \frac{-43 + 48}{1} = 5$$

$$l_{33} = \sqrt{98 - (-8)^2 - 5^2} = \sqrt{98 - 64 - 25} = \sqrt{9} = 3$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{bmatrix}$$

Then:

$$A = LL^T = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{bmatrix}\begin{bmatrix} 2 & 6 & -8 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$$

Example 2:
Let:

$$A = \begin{bmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{bmatrix}$$

Step 1: Check Symmetry and Positive Definite
A symmetry and positive definite
Step 2: Compute Cholesky Decomposition

$$l_{11} = \sqrt{25} = 5$$

$$l_{21} = \frac{15}{5} = 3, \quad l_{31} = \frac{-5}{5} = -1$$

$$l_{22} = \sqrt{18 - 3^2} = \sqrt{9} = 3$$

$$l_{32} = \frac{-0 - (-1)(3)}{3} = \frac{-0 + 3}{3} = 1$$

$$l_{33} = \sqrt{11 - (-1)^2 - 1^2} = \sqrt{11 - 1 - 1} = \sqrt{9} = 3$$

2.2.2. Application of Cholesky decomposition method

1. Solving Linear Systems: For $Ax = b$, compute L from $A = LL^T$, then solve $Ly = b$ and $L^T x = y$.
2. Least Squares Optimization: Used in solving normal equations of the form $A^T Ax = A^T b$.
3. Monte Carlo Simulations: Used to generate samples from multivariate normal distributions.
4. Kalman Filters: Cholesky decomposition improves numerical stability when estimating covariance matrices.

Summary

- Cholesky decomposition is a matrix factorization for symmetric, positive definite matrices.
- It expresses A as $A = LL^T$ where L is lower triangular with positive diagonal entries.
- It is more efficient and numerically stable than LU decomposition for such matrices.
- Widely used in numerical analysis, statistics, and optimization problems.

III. ANALYSIS AND RESULT

3.1 Theoretical Investigation into the Numerical Stability and Accuracy of Matrix Decomposition Methods

Matrix decomposition techniques play a pivotal role in numerical linear algebra, underpinning a wide range of computational methods in science and engineering. We presents a theoretical examination of the numerical stability and accuracy of matrix decomposition methods: Cholesky decomposition, and singular value decomposition (SVD). By exploring each method's mathematical structure, sensitivity to perturbations, and inherent stability characteristics, we aim to elucidate the conditions under which each technique excels or deteriorates in practical computation.

3.2Singular Value Decomposition (SVD)

SVD represent any $m \times n$ matrix A as $A = U\Sigma V^T$, where U and V are orthogonal matrices and Σ is a diagonal matrix containing the singular values. It is a powerful and general decomposition, used in least squares problems, pseudo inverse computation, and data compression.

3.2.1Numerical Stability of SVD

SVD is among the most stable of all matrix decomposition. The orthogonality of U and V ensures that norm-preserving transformations are

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used throughout the process, thereby minimizing the propagation of errors.

SVD is robust even for ill-conditioned or rand-deficient matrices, making it particularly useful when other decomposition fail.

3.2.2Numerical Accuracy of SVD

SVD provides excellent numerical accuracy, especially in problems involving low-rank approximations or noisy data. The singular values indicate the intrinsic dimensionality of the data, and small singular values can be safely truncated to reduce noise.

Backward and forward errors are typically minimal, and the method produces the best low-rank approximation of a matrix in terms of the Frobenius and 2-norms.

3.3 Cholesky Decomposition

Cholesky decomposition is a specialized form of LU decomposition for symmetric, positive definite matrices. It factors A as $A = LL^T$, where L is a lower triangular matrix with positive diagonal entries.

3.3.1 Numerical Stability of Cholesky Decomposition

Cholesky decomposition is numerically stable if the matrix is well-conditioned and satisfies the positive definiteness requirement. Since the method involves square roots of the diagonal elements, numerical issues can arise if the matrix is nearly singular or not strictly positive definite.

Unlike general LU decomposition, Cholesky does not require pivoting for stability, but its application is limited to a specific class of matrices.

3.3.2Numerical Accuracy of Cholesky Decomposition

Cholesky decomposition is backward stable under its domain of applicability. It produces results with high accuracy, benefiting from fewer arithmetic operations compared to general LU or QR decompositions. However, its performance can deteriorate rapidly when applied to matrices that deviate from symmetry or positive definiteness.

3.4Comparative Analysis

The table below summarizes key aspect of stability and accuracy for each method:

Method	Pivoting required	Stability	Accuracy
Singular Value Decomposition	No	Very High	Very High
Cholesky	No	High	High

The theoretical investigation underscores that no single decomposition method is universally optimal. Cholesky decomposition is extremely efficient and accurate within its domain but limited in scope while SVD stands out for its robustness and accuracy, particularly in ill-posed or data-sensitive problems.

In choosing a decomposition method, one must weigh stability, accuracy, computational cost, and the structural properties of the matrix. Understanding the numerical behavior of these algorithm ensures more reliable and informed application in mathematical modeling and scientific computing.

3.5 Theoretical illustration of numerical Stability of SVD and Cholesky Decomposition Methods.

Here, we present solved examples for Singular Value Decomposition (SVD) and Cholesky Decomposition. Each example includes forward and backward error analysis to assess numerical stability and accuracy. A concluding section compares computational speed and robustness across methods.

3.5.1Singular Value Decomposition (SVD)

Example: Low-Rank Approximation

Let

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \Rightarrow \lambda = 16{,}4 \Rightarrow \sigma = 4{,}2$$

SVD:

$$A = U\Sigma V^T, \Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

Low-rank approximation using only $\sigma_1 = 4$

$$A_1 = \sigma_1 u_1 v_1^T$$

Conclusion: SVD is numerically robust, even under noise or Ill-conditioning.

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3.5.2Cholesky Decomposition
Example: Symmetric Positive Definite Matrix
Let

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Cholesky

$$L = \begin{bmatrix} 2 & 0 \\ 1 & \sqrt{2} \end{bmatrix}, \; LL^T = A$$

Solving gives:

$$\breve{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Error Analysis:

- Forward error: $\|x - \breve{x}\| = 0$
- Residual: $r = b - A\hat{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- All errors are zero

3.6 Comparative Applications of SVD and Cholesky decomposition methods in solving various cases of linear equations.

Here, we compare the SVD and Cholesky matrix decomposition techniques in solving various cases of linear systems. For each method, we apply it to two representative systems it is best suited for and provide a complete, worked example with explicit formulation of the system, augmented matrix (if applicable), and step-by-step solution. Accuracy and stability are assessed to determine the approach for different system structures.

3.6.1 SVD: Rank-Deficient System
Example 1.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1.0001 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 2.0001 \end{bmatrix}$$

$$\sigma_1 = 2.00005, \;\; \sigma_2 = 0.00005 \Rightarrow x = V\Sigma^{-1}U^T b$$

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Example 2.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$x = V\Sigma^{+}U^T b \Rightarrow x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3.6.2 Cholesky Decomposition: Symmetric Positive Definite System
Example1.
System:

$$4x + 2y = 6{,}2x + 3y = 5$$

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}, \; b = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$L = \begin{bmatrix} 2 & 0 \\ 1 & \sqrt{2} \end{bmatrix}$$

$$Ly = b \Rightarrow y = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \;\; L^T x = y \Rightarrow x = \begin{bmatrix} 1 \\ 0.7071 \end{bmatrix}$$

Example 2.
System:

$$9x + 3y = 12, \quad 3x + 4y = 10$$

$$A = \begin{bmatrix} 9 & 3 \\ 3 & 4 \end{bmatrix}, \; b = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$$L = \begin{bmatrix} 3 & 0 \\ 1 & \sqrt{3} \end{bmatrix}, \quad x \approx \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Remark:
Each decomposition method has a best case scenario for linear system:

- Cholesky: Symmetric positive definite problems
- SVD: ill-conditioned or rand deficient systems.

IV. SUMMARY, RECOMMENDATION AND CONCLUSION.

4.1 Summary:

This study undertook a comparative analysis of two matrix decomposition methods- Cholesky decomposition and Singular Value Decomposition with the goal of evaluating their performance in solving large scale linear systems. Each method was analyzed in terms of numerical of numerical stability, computational efficiency, and suitability for different classes of problems.

The result showed that Cholesky decomposition emerged as the most efficient in terms of speed and memory usage but is limited to symmetric positive-definite matrices. Within its scope, however, it consistently outperformed LU and QR in computational efficiency. Applications in multivariate statistics, covariance analysis, and quadratic programming highlight its value in both theory and practice.

Singular Value Decomposition (SVD) was the most versatile and robust of all the methods studied. It is particularly valuable for ill-conditioned or rank-deficient problems, ensuring stable solutions where other method fail. However, its strength comes at a high computational cost, making it unsuitable for very large scale systems unless approximations such as truncated SVD are applied. Applications in machine learning, recommender systems, and dimensionality reduction underscore its practical significance.

Overall, the comparative study showed that no single decomposition method is universally superior. Each has advantages and limitations depending on the structure and conditioning of the matrix. For large-scale and sparse systems, hybrid approaches such as combining decomposition with iterative solvers offer a balanced solution, leveraging both stability and efficiency.

4.2 Recommendation:

1. Use Cholesky decomposition when working with symmetric positive definite matrices. It is most efficient and stable choice in such cases.
2. Use SVD for solving ill-conditioned or rank-deficient problems where accuracy is critical, such as in signal processing, machine learning, or inverese problems.

4.3Conclusion:

There is no one size solution when it comes to solving large scale systems of equations using matrix decomposition. Each method has its advantages depending on the nature of the matrix and the requirements of the problem. Cholesky are fast and effective for structured. Well-conditioned systems, while SVD offer greater stability for ill-conditioned or over determined systems. The best approach is to first analyze the structure and condition of the matrix. Based on this, select the decomposition method that offers the best trade off between speed, stability, and accuracy. In practice, hybrid approaches such as combining decomposition with iterative refinement or regularization can provide even better performance, particularly for large scale problems encountered in scientific computing, engineering, and data intensive applications.

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