

Analysis Of Information Measures in Communication Engineering

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Abstract- Our present work deals with information measures involving two probability distributions. Our focus is more on these information measures and generalizations. Some of the specific applications and interpretations of these information measures will discuss in this paper

Index Terms- Information, Csiszár's F-Divergence, Chi-Square Divergence

I. INTRODUCTION: CSISZÁR'S F-DIVERGENCE AND PROPERTIES

The Csiszár's f -divergence is a general class of divergence measures that includes several divergences used in measuring the distance or affinity between two probability distributions. This class is introduced by using a convex function f , defined on $(0, \infty)$. An important property of this divergence is that many known divergences can be obtained from this measure by appropriately defining the convex function f .

For a convex function $f : [0, \infty) \rightarrow \mathbf{R}$, the f -Divergence measure of the probability distributions P and Q , defined by Csiszár's [24, 25] and Ali & Silvey [3], is given by

$$C_f(P, Q) = \sum_{i=1}^n q_i f\left(\frac{p_i}{q_i}\right)$$

Most common choice is of satisfy $f(1) = 0$, so that $C_f(P, Q) = 0$. Convexity ensures that the divergence measure $C_f(P, Q)$ is always non-negative.

Many authors introduced several divergence measures. These divergences are very useful in information theory for comparing discrete probability distributions.

In this section, we have listed different measures of information proposed by various researchers in associated fields:

1. Kullback-Leibler distance (Kullback and Leibler)

$$K(P, Q) = \sum_{i=1}^n p_i \log \frac{p_i}{q_i}$$

2. Chi-square divergence or Pearson divergence (Pearson)

$$\chi^2(P, Q) = \sum_{i=1}^n \frac{(p_i - q_i)^2}{q_i} = \sum_{i=1}^n \frac{p_i^2}{q_i} - 1$$

3. Relative Jensen-Shannon divergence (Sibson)

$$F(P, Q) = \sum_{i=1}^n p_i \log \left(\frac{2p_i}{p_i + q_i} \right)$$

4. Relative Arithmetic- Geometric divergence (Taneja)

$$G(P, Q) = \sum_{i=1}^n \left(\frac{p_i + q_i}{2} \right) \log \left(\frac{p_i + q_i}{2p_i} \right)$$

5. Hellinger discrimination (Hellinger)

$$h(P, Q) = 1 - B(P, Q) = \frac{1}{2} \sum_{i=1}^n (\sqrt{p_i} - \sqrt{q_i})^2$$

$$\text{Where } B(P, Q) = \sum_{i=1}^n \sqrt{p_i q_i}$$

is known as Bhattacharyya divergence measure.

6. Triangular discrimination (Dacunha- Castelle etc. all)

$$\Delta(P, Q) = 2[1 - H(P, Q)] = \sum_{i=1}^n \frac{(p_i - q_i)^2}{p_i + q_i}$$

$$\text{Where } H(P, Q) = \sum_{i=1}^n \frac{2p_i q_i}{p_i + q_i}$$

is known as harmonic mean divergence measure

7. Symmetric Chi-square divergence (Dragomir etc. all)

$$\psi(P, Q) = \chi^2(P, Q) + \chi^2(Q, P) = \sum_{i=1}^n \frac{(p_i - q_i)^2 (p_i + q_i)}{p_i q_i}$$

8. J-divergence measure (Kullback and Leibler)

$$J(P, Q) = K(P, Q) + K(Q, P) = J_R(P, Q) + J_R(Q, P) = \sum_{i=1}^n (p_i - q_i) \log \frac{p_i}{q_i}$$

9. Relative J-divergence measure (Dragomir etc. all)

$$J_R(P, Q) = \sum_{i=1}^n (p_i - q_i) \log \frac{p_i + q_i}{2q_i}$$

10. Arithmetic-Geometric mean divergence (Taneja)

$$T(P, Q) = \frac{1}{2} [G(P, Q) + G(Q, P)] = \sum_{i=1}^n \left(\frac{p_i + q_i}{2} \right) \log \left(\frac{p_i + q_i}{2\sqrt{p_i q_i}} \right)$$

Where $G(P, Q)$ is the Relative AG divergence.

II. NEW ANALYSIS

1. In above measures 1, 2, 3, 4 and 9 are non-symmetric information measures and 5, 6, 7, 8, 9 & 10 are symmetric information measures.
2. All above information measures are very useful to calculate affinity and distance between to discrete probability distributions.
3. All information measures are of Csiszár's f -divergence class.
4. All functions corresponding to these information measures are convex and normalized.
5. Applications of these measures are very useful in all communication systems.

III. CONCLUSION

We analyzed above all information measure, and then presented our five results. These five characteristics are very useful to young researchers, using these, they can evaluate more results for information measures.

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