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# Analysis Of Damped Mechanical Systems Using Differential Transform Method and Laplace Transform

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***Abstract-* This study examines the effect of damping on a mechanical system modelled by a second-order differential equation using the Differential Transform Method (DTM) and the Laplace Transform. The DTM is applied as the primary technique to obtain a power series solution, while the Laplace Transform provides an exact analytical solution for validation. Numerical results show that the DTM solution converges rapidly and agrees closely with the exact solution. The findings confirm that damping significantly reduces system motion and ensures a stable equilibrium, particularly under critical damping conditions.***

***Keywords: differential transform method; laplace transform, damped mechanical system, system stability***

## I. INTRODUCTION

Damping plays a major role in determining how the system behaves over time. It reduces the amplitude of oscillation and represents the mechanism through which energy is dissipated, leading to a gradual reduction in oscillatory motion. The effect of damping on a mechanical system is seen in the way it controls vibration and improves system stability. When damping is small, the system continues to oscillate for a long time with slowly decreasing amplitude. When damping is increased, the oscillations reduce more quickly. At a particular level known as critical damping, the system returns to equilibrium in the shortest time without oscillating. When damping is very large, the system returns slowly to equilibrium without oscillation. These different damping conditions significantly affect the nature of the solution of the governing differential equation. From a mathematical standpoint, the study of damping involves the formulation and solution of second-order ordinary differential equations. These equations describe how system parameters such as mass, stiffness, and damping coefficient affect the motion of a system over time.

## II. REVIEW OF RELATED WORK

Recent discussions continue to show that damping remains a central idea in understanding mechanical systems because it connects mathematical description with observable behavior in a very direct way. Pinsky (2018) explains that damped motion provides a useful example of how mathematical models can describe change, decay, and eventual stability within a single system. Olsder and van der Woude (2019) also note that damping introduces an important balancing effect in dynamic systems by preventing persistent uncontrolled motion and guiding the system toward rest. Dutta (2021) further explains that modern studies of mechanical systems continue to pay close attention to damping because it strongly influences the overall behavior of the system and cannot be ignored in serious analysis. Kiusalaas (2022) adds that damped mechanical systems remain valuable in both theory and application because they help to show how real systems respond over time under the influence of resistance. Altintas (2023) also observes that the study of damping is still relevant in current work on motion and vibration because it explains why actual mechanical response differs from perfectly idealized behavior. Taken together, these views show that damping is essential for understanding the true nature of mechanical systems, since it explains why motion weakens, why equilibrium is eventually reached, and why mathematical models of real mechanical systems must take energy loss into account if they are to give a meaningful description of system behavior.

The Differential Transform Method is an important technique used in solving differential equations, particularly those that arise in the study of mechanical systems, because it provides a simple and systematic way of obtaining approximate solutions without requiring complicated calculations. Zhou (2000) introduces the Differential Transform Method as a procedure that converts differential equations into a sequence of simpler algebraic relations, making it easier to construct solutions step by step. Zill (2001) explains that many mechanical systems are described by differential equations, and therefore any method that simplifies such equations is valuable in understanding system behavior over time. Arikoglu and Ozkol (2006) further demonstrate that the Differential Transform Method can be applied effectively to problems involving mechanical motion, especially when traditional analytical methods become difficult to use. Boylestad (2006) also notes that simplifying complex mathematical expressions is important when studying systems that involve repeated motion and changing conditions. Simmons (2007) emphasizes that solving differential equations is central to understanding how mechanical systems behave, and he points out that alternative methods can provide additional ways of approaching such problems. Ayaz (2008) explains that the Differential Transform Method works by breaking down complex equations into smaller parts that can be solved progressively, making it easier to follow the pattern of motion in a system. Thomson (2008) adds that methods which simplify motion analysis are particularly useful in mechanical systems where continuous change occurs over time.

The usefulness of the Differential Transform Method becomes clearer when dealing with systems that are influenced by damping and other factors that make their behavior more complicated. Meirovitch (2010) explains that mechanical systems often involve interactions that make their mathematical representation difficult, and therefore require solution methods that can manage this complexity effectively. Kreyszig (2011) similarly notes that many differential equations cannot be solved easily using standard techniques, which makes alternative approaches such as the Differential Transform Method especially useful. Boyce and DiPrima (2012) further explain that approximate methods are often necessary when exact solutions are not readily available, particularly in real mechanical systems. Hassan (2012) demonstrates that the Differential Transform Method can produce accurate approximate solutions for a wide range of problems, including those involving mechanical systems with varying conditions. Ross (2014) also explains that such methods help in understanding how solutions behave even when exact formulas are difficult to obtain. Inman (2014) adds that efficient methods for analyzing motion are important in studying mechanical systems, especially those involving vibration and damping effects.

## III. DEFINITION AND OPERATIONAL FOUNDATION OF DTM

The Differential Transform Method (DTM) is a semi-analytical technique developed for solving ordinary and partial differential equations. It is fundamentally based on the Taylor series expansion of analytic functions, but it differs from the classical Taylor method in that it eliminates the direct computation of higher-order derivatives. Instead, DTM transforms differential equations into algebraic recurrence relations, thereby simplifying the computational procedure while preserving analytical structure.

Let x(t) be an analytic function in a neighborhood of t=0. The differential transform of x(t) is defined as

$X(k) = \frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}$, k=1,2,3,…

where X(k)represents the transformed function and corresponds to the coefficient of the k-th order term in the series expansion of x(t).

The inverse differential transform reconstructs the original function as a power series:

$$x(t) = \sum_{k=0}^{\infty} X(k) t^k$$

*A. Differential Transform of Simple Functions*

In this section, several basic functions are considered, and their differential transforms are derived directly from the definition of DTM. Below are the Differential Transforms of some simple functions:

i) *Exponential Function*

Consider

$$x(t)=e^{at},$$

where a is a constant.

Compute successive derivatives:

$$\frac{d}{dt}(e^{at}) = ae^{at},$$
$$\frac{d^2}{dt^2}(e^{at}) = a^k e^{at}.$$

Evaluating at $t = 0$,

$$[e^{at}]_{t=0} = 1.$$

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Thus,

$$X(k) = \frac{1}{k!} a^k.$$

Hence the inverse transform gives

$$e^{at} = \sum_{k=0}^{\infty} \frac{a^k}{k!} t^k.$$

This confirms that DTM naturally reproduces the Maclaurin series of the exponential function.

ii) *Sine Function*

Consider

$$x(t) = \sin(at).$$

The derivatives follow a cyclic pattern:

$\sin(at)$,a $\cos(at)$,$-a^2\sin(at)$,$-a^3\cos(at)$,$a^4 \sin(at)$ ,…….

Evaluating at $t = 0$:

$$\sin 0 = 0$$
$$\cos 0 = 1$$

Thus:

Even- order derivatives evaluated at 0 are zero, while odd-order derivatives alternate in sign.

Therefore,

$$X(k) = \begin{cases} (-1)^m \dfrac{a^{2m+1}}{(2m+1)!}, & k = 2m+1, \\ 0, & k = 2m. \end{cases}$$

Thus the inverse Transform yields,

$$\sin at = \sum_{m=0}^{\infty} (-1)^m \frac{a^{2m+1}}{(2m+1)!} t^{2m+1}.$$

iii) *Cosine Function*

Consider

$x(t) = \cos(at)$.

The derivatives follow the cycle:

$\cos(at)$ , $-$a $\sin(at)$ , $-a^2\cos(at)$ , $a^3\sin(at)$ , $a^4\cos(at)$ , ..

Evaluating at $t = 0$:

$$\cos(0) = 1,$$
$$\sin(0) = 0$$

Thus,

$$X(k) = \begin{cases} (-1)^m \dfrac{a^{2m}}{(2m)!}, k = 2m \\ 0, \quad k = 2m+1. \end{cases}$$

Hence,

$$\cos(at) = \sum_{m=0}^{\infty} (-1)^m \frac{a^{2m}}{(2m)!} t^{2m}$$

iv) *First Derivative*

Let x(t) be analytic at t=0, and let its differential transform be defined as:

$$X(k) = \frac{1}{k!} \left[\frac{d^k x(t)}{dt^k}\right]_{t=0}$$

We now determine the transform of the first derivative x′(t).

By definition, the differential transform of x′(t) is:

$$\mathcal{D}\{x'(t)\} = \frac{1}{k!} \left[\frac{d^k}{dt^k}\big(x'(t)\big)\right]_{t=0}$$

Observe that differentiating x′(t) k times is equivalent to differentiating x(t) k+1 times:

$$\frac{d^k}{dt^k}\big(x'(t)\big) = \frac{d^{k+1}x(t)}{dt^{k+1}}$$

Thus,

$$\mathcal{D}\{x'(t)\} = \frac{1}{k!} \left[x^{(k+1)}(t)\right]_{t=0}$$

From the definition of the transform of x(t):

$$X(k+1) = \frac{1}{(k+1)!} \left[x^{(k+1)}(t)\right]_{t=0}$$

Solving for the derivative term:

$$x^{(k+1)}(0) = (k+1)!\, X(k+1)$$

Substituting into the earlier expression:

$$\mathcal{D}\{x'(t)\} = \frac{1}{k!} (k+1)!\, X(k+1)$$

Simplifying:

$$\frac{(k+1)!}{k!} = (k+1)$$

Hence,we obtain:

$$\mathcal{D}\{x'(t)\} = (k+1)X(k+1)$$

v) *Second Derivative*

We now determine the transform of the second derivative x″(t).

By definition:

$$\mathcal{D}\{x''(t)\} = \frac{1}{k!} \left[\frac{d^k}{dt^k}\big(x''(t)\big)\right]_{t=0}$$

Observe that differentiating x″(t) k times is equivalent to differentiating x(t) k+2 times:

$$\frac{d^k}{dt^k}\big(x''(t)\big) = \frac{d^{k+2}x(t)}{dt^{k+2}}$$

Thus,

$$\mathcal{D}\{x''(t)\} = \frac{1}{k!} \left[x^{(k+2)}(t)\right]_{t=0}$$

From the transform definition:

$$X(k+2) = \frac{1}{(k+2)!} \left[x^{(k+2)}(t)\right]_{t=0}$$

Hence,

$$x^{(k+2)}(0) = (k+2)!\, X(k+2)$$

Substituting:

$$\mathcal{D}\{x''(t)\} = \frac{1}{k!} (k+2)!\, X(k+2)$$

Now simplify the factorial ratio:

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$$\frac{(k+2)!}{k!} = (k+2)(k+1)$$

Therefore:

$$\mathcal{D}\{x''(t)\} = (k+2)(k+1)X(k+2)$$

vi) *Nth - Derivative*

From the derivations above, we observe a pattern. For higher-order derivatives:

1) The index shifts forward by the order of differentiation.
2) The coefficient is multiplied by a polynomial expression in k.

In general, for the n-th derivative:

$$\mathcal{D}\{x^{(n)}(t)\} = \frac{(k+n)!}{k!}X(k+n)$$

This can also be written as:

$$\mathcal{D}\{x^{(n)}(t)\} = (k+1)(k+2)\cdots(k+n)X(k+n)$$

*B. Laplace Transform*

This transformation greatly simplifies the process of solving linear differential equations because differentiation in the time domain becomes multiplication in the transform domain.

Let f(t) be a function defined for $t \geq 0$. The Laplace Transform of f(t) is defined as:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\ dt$$

provided the integral converges.

The Laplace Transform is linear. That is, for functions f(t), g(t), and constants a, b:

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

This property allows differential equations to be transformed term-by-term.

*C. Laplace Transform of Simple Functions*

In order to apply the Laplace Transform effectively to differential equations, it is essential to understand how it operates on elementary functions. Many solutions of linear differential equations involve constants, polynomials, exponential functions, and trigonometric functions. Therefore, deriving their Laplace transforms from first principles provides the operational foundation necessary for later applications.

Recall that the Laplace Transform of a function f(t) is defined as:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\ dt$$

provided the integral converges.

We now compute the transforms of several basic functions step by step.

i) *Exponential Function*

Consider

$$f(t) = e^{at}$$

Substituting into the definition:

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at} dt$$

Combine the exponential:

$$e^{-st} e^{at} = e^{-(s-a)t}$$

Thus:

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-(s-a)t} dt$$

Evaluating:

$$\int_0^\infty e^{-(s-a)t} dt = \left[-\frac{1}{s-a} e^{-(s-a)t}\right]_0^\infty$$

Assuming s>a

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

Hence:

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \quad s > a$$

ii) *Sine Function*

Consider:

$$f(t) = \sin(at)$$

Using Euler's Identity:

$$\sin(at) = \frac{e^{iat} - e^{-iat}}{2i}$$

Applying linearity of Laplace transform:

$$\mathcal{L}\{\sin(at)\} = \frac{1}{2i}\left(\frac{1}{\mathrm{s} - \mathrm{ia}} - \frac{1}{s + ia}\right)$$

Simplifying the expression:

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$$

Thus:

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$$

iii) *Cosine Function*

Consider:

$$f(t) = \cos(at)$$

Using Euler's identity:

$$\cos(at) = \frac{e^{iat} + e^{-iat}}{2}$$

Applying the Laplace transform:

$$\mathcal{L}\{\cos(at)\} = \frac{1}{2}\left(\frac{1}{s-ia} + \frac{1}{s+ia}\right)$$

Simplifying:

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$$

Thus:

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}$$

iv) *Laplace Transform of First Derivative*

A crucial property of the Laplace Transform is its treatment of derivatives.

Let:

$$\mathcal{L}\{f(t)\} = F(s)$$

We compute:

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\ dt$$

Using integration by parts.

Let

$$u = e^{-st}, \quad dv = f'(t)\, dt$$

Then

$$du = -se^{-st}\, dt, \quad v = f(t)$$

Applying integration by parts:

$$\int_0^\infty e^{-st} f'(t)\ dt = [e^{-st} f(t)]_0^\infty + s\int_0^\infty e^{-st} f(t)\ dt$$

Assuming f(t) does not grow faster than an exponential, we have:

$$\lim_{t\to\infty} e^{-st} f(t) = 0$$

Thus:

$$[e^{-st} f(t)]_0^\infty = -f(0)$$

Therefore:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

v) *Laplace Transform of the Second Derivative*

Using the first-derivative property:

$$\mathcal{L}\{f''(t)\} = s\mathcal{L}\{f'(t)\} - f'(0)$$

Substitute:

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

Hence:

$$\mathcal{L}\{f''(t)\} = s\,(\,sF(s) - f(0)\,) - f'(0)$$

Simplifying:

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

In general, for the n-th derivative:

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$

*D. Application of Differential Transform Method to a First Order Differential Equation*

To illustrate the practical implementation of the Differential Transform Method (DTM), we apply it to the first-order linear differential equation:

$$x'(t) + 3x(t) = 0$$

Subject to the initial condition:

$$x(0) = 2$$

Since the Differential Transform Method is based on Taylor series expansion about t=0, we express the unknown function as a power series centered at the origin. This allows the differential equation to be converted into an algebraic recurrence relation for the transform coefficients.

Thus, we write

$$x(t) = \sum_{k=0}^{\infty} X(k)t^k$$

By definition of DTM,

$$X(k) = \frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}$$

From the given initial condition, the first coefficient is immediately obtained as

$$X(0) = x(0) = 2$$

To transform the differential equation, we use the DTM rule for the first derivative:

$$\mathcal{D}\{x'(t)\} = (k+1)X(k+1)$$

Applying the transform to each term of the equation gives

$$(k+1)X(k+1) + 3X(k) = 0$$

This algebraic relation holds for

$$k = 0,1,2,\dots$$

We now solve for the next coefficient in terms of the previous one:

$$X(k+1) = -\frac{3}{k+1}X(k)$$

This recurrence relation enables us to compute all coefficients sequentially.

Starting with

$$X(0) = 2$$

We obtain

$$X(1) = -6$$
$$X(2) = 9$$
$$X(3) = -9$$
$$X(4) = \frac{27}{4}$$

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$$X(5) = -\frac{81}{20}$$
$$X(6) = \frac{81}{40}$$
$$X(7) = -\frac{243}{280}$$
$$X(8) = \frac{729}{2240}$$
$$X(9) = -\frac{243}{2240}$$
$$X(10) = \frac{729}{22400}$$

From the pattern above, the general expression for the coefficients is

$$X(k) = \frac{2(-3)^k}{k!}$$

Using these coefficients, the truncated DTM approximation for k=0 to 10 becomes

$$x_{10}(t) = 2 - 6t + 9t^2 - 9t^3 + \frac{27}{4}t^4 - \frac{81}{20}t^5 + \frac{81}{40}t^6 - \frac{243}{280}t^7 + \frac{729}{2240}t^8 - \frac{243}{2240}t^9 + \frac{729}{22400}t^{10}$$

Substituting values of t from 0.01 to 0.10 gives

| T | $x_{10}(t)$ |
|---|---|
| 0.01 | 1.9408910671 |
| 0.02 | 1.8835290672 |
| 0.03 | 1.8278623650 |
| 0.04 | 1.7738408734 |
| 0.05 | 1.7214159529 |
| 0.06 | 1.6705404228 |
| 0.07 | 1.6211684958 |
| 0.08 | 1.5732557254 |
| 0.09 | 1.5267590388 |
| 0.1 | 1.4816364414 |

*E. Application of Laplace Transform to a First Order Differential Equation*

We consider the initial value problem

$$x'(t) + 3x(t) = 0$$

with the initial condition

$$x(0) = 2$$

The Laplace Transform method is applied because it converts differentiation in the time domain into algebraic operations in the transform domain. This significantly simplifies the solution of linear differential equations with given initial conditions.

Let the Laplace Transform of x(t) be denoted by

$$\mathcal{L}\{x(t)\} = X(s)$$

Using the standard derivative property of the Laplace Transform, we have

$$\mathcal{L}\{x'(t)\} = sX(s) - x(0)$$

Applying the Laplace Transform to the differential equation term by term yields

$$\mathcal{L}\{x'(t)\} + 3\mathcal{L}\{x(t)\} = 0$$

Substituting the transform formulas gives

$$sX(s) - x(0) + 3X(s) = 0$$

Since the initial condition is

$$x(0) = 2$$

we obtain

$$sX(s) - 2 + 3X(s) = 0$$

Collecting terms involving X(s), we get

$$(s + 3)X(s) - 2 = 0$$

Rearranging,

$$(s + 3)X(s) = 2$$

and solving for X(s),

$$X(s) = \frac{2}{s + 3}$$

Thus, the differential equation has been transformed into a simple algebraic expression in the variable s.

To recover the solution in the time domain, we apply the inverse Laplace Transform. Using the standard result

$$\mathcal{L}^{-1}\left\{\frac{1}{s + a}\right\} = e^{-at}$$

We obtain

$$x(t) = 2e^{-3t}$$

This represents the exact analytical solution of the initial value problem.

For numerical evaluation, we compute

$$x(t) = 2e^{-3t}$$

For values of;

$$t = 0.01 \text{ to } 0.10$$

The results are shown below:

| T | $x(t) = 2e^{-3t}$ |
|---|---|
| 0.01 | 1.9408910671 |
| 0.02 | 1.8835290672 |
| 0.03 | 1.8278623650 |
| 0.04 | 1.7738408734 |

| T | $x(t) = 2e^{-3t}$ |
|---|---|
| 0.05 | 1.7214159529 |
| 0.06 | 1.6705404228 |
| 0.07 | 1.6211684958 |
| 0.08 | 1.5732557254 |
| 0.09 | 1.5267590388 |
| 0.1 | 1.4816364414 |

*F. Comparing Differential Transform Method and Laplace Transform Method*

The Differential Transform Method produced the truncated series solution

$$x_{10}(t) = \sum_{k=0}^{10} \frac{2(-3)^k}{k!} t^k$$

while the Laplace Transform method yielded the exact analytical solution

$$x(t) = 2e^{-3t}$$

A numerical comparison on the interval

$$0.01 \leq t \leq 0.10$$

shows that the truncated DTM solution agrees extremely closely with the exact Laplace solution. The negligible difference between the two results demonstrates the rapid convergence of the power series representation and confirms the accuracy and reliability of the Differential Transform Method for solving linear differential equations, particularly for small values of t.

*1) Table Showing DTM and Laplace Transform of Some Functions*

| Functions | DTM | Laplace Transform |
|---|---|---|
| $f(t) = 1$ | $X(0) = 1$<br>$X(k) = 0, \quad k \geq 1$ | $\mathcal{L}\{1\} = \frac{1}{s}$ |
| $f(t) = t^n$ | $X(n) = 1$<br>$X(k) = 0, \quad k \neq n$ | $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ |
| $f(t) = e^{at}$ | $X(k) = \frac{a^k}{k!}$ | $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$ |
| $f(t) = \sin(at)$ | $\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$ | $\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$ |
| $f(t) = \cos(at)$ | $\mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2}$ | $\mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2}$ |

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