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Trigonometric Near-Rings as a Unified Algebraic Framework for Quantum Computation

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Abstract- We introduce a unified algebraic framework for quantum computation based on finite trigonometric near-rings. Motivated by unification ideas in physics, the proposed structure connects trigonometric functions, non-commutative algebra, and quantum gate operations within a single finite system. By constructing near-rings generated by discrete phase operators and the Hadamard operator, we show that quantum phase interference and basis transformations naturally give rise to non-commutative near-ring structures. The developed framework generalizes earlier Heisenberg-type matrix near-rings to a trigonometric setting, providing a discrete algebraic model for quantum circuits involving phase and Clifford gates. Explicit finite examples including a non-commutative near-ring of order sixteen are presented to illustrate the theory. This approach offers a conceptually simple yet mathematically rigorous bridge between trigonometry and quantum computation and highlights the relevance of near-ring theory as a foundational tool for modeling finite quantum systems.

Index Terms- Trigonometric Near-Rings, Quantum Computing; Non-Commutative Algebra, Hadamard Gate, Phase Gates, Finite Near-Rings, Heisenberg Near-Rings, Quantum Circuits, Algebraic Unification.

I. INTRODUCTION

Unification has long been a central theme in both mathematics and physics. In physics, unification programs aim to describe diverse fundamental interactions within a single theoretical framework. In mathematics, similar unifying structures arise when apparently distinct concepts—such as geometry, algebra, and analysis—are revealed to be manifestations of a deeper algebraic principle.

Quantum computation, which inherently combines linear algebra, complex analysis, and discrete structures, provides a natural setting for such unification. Trigonometric functions, particularly sine, cosine, and complex exponentials, play a fundamental role in quantum mechanics through phase interference and unitary evolution. At the same time, the non-commutativity of quantum operators lies at the heart of quantum behavior, as exemplified by the Heisenberg uncertainty principle.

Near-ring theory offers a flexible algebraic framework capable of modeling non-commutative systems that fall outside the scope of classical ring theory. Finite near-rings constructed from matrix and operator systems have previously been used to model discrete versions of quantum mechanical structures. In particular, Heisenberg-type near-rings capture non-commutativity in a finite algebraic setting.

The purpose of this paper is to introduce and study finite trigonometric near-rings as a unifying algebraic model for quantum computation. By combining discrete trigonometric phase operators with basis-changing transformations such as the Hadamard operator, we obtain finite non-commutative near-rings that naturally encode the composition of quantum gates. This construction extends earlier Heisenberg near-ring models and provides a direct algebraic interpretation of quantum circuits involving phase and Clifford gates.

II. FINITE TRIGONOMETRIC NEAR-RINGS

Let $\omega = e^{\{i\pi/4\}}$. Define $N_{16} = \{ \omega^{k\,I}, \omega^{k\,H} \mid k \in \mathbb{Z}_8 \}$. Under operator superposition and matrix multiplication, N_{16} forms a finite right near-ring of order 16. Explicit multiplication rules and closure are

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verified.

III. HEISENBERG GROUP OVER $\mathbb{Z}_2$

Elements are triples (a, b, c), a, b, c $\in \mathbb{Z}_2$, with additive group operation: $(a, b, c) \oplus (a', b', c') = (a + a', b + b', c + c' + ab') \pmod 2$.

This group is non-abelian. Labeling the elements gives eight elements corresponding to the discrete Heisenberg group.

The identity element is e = (0,0,0). Every element has order 2. The group center is $Z(H_3(\mathbb{Z}_2)) = \{e, z\}$. When combined with matrix multiplication, this forms a finite near-ring of order 8.

IV. PAULI MATRICES AND QUANTUM INTERPRETATION

Pauli matrices are: I = [[1,0],[0,1]], X = [[0,1],[1,0]], Y = [[0,-i],[i,0]], Z = [[1,0],[0,-1]].

Key relations: $X^2 = Z^2 = I$, $XZ = -ZX$, $Y = iXZ$. Ignoring global phase ±1, ±i, these generate the single-qubit Pauli group.

Define the map $\Phi : H_3(\mathbb{Z}_2) \to P_1 / \{\pm I\}$ by $\Phi(a,b,c) = i^c X^a Z^b$. This is a homomorphism since $\Phi(u \oplus v) = \Phi(u)\Phi(v)$. The additive non-commutativity corresponds to quantum phase.

The central element z = (0,0,1) corresponds to the phase i. Thus, $H_3(\mathbb{Z}_2) \cong$ Pauli group modulo phase.

Near-ring viewpoint: addition corresponds to the Heisenberg additive law, multiplication corresponds to operator composition. The structure is non-abelian and right distributive, but not a ring, reflecting quantum behavior.

V. TWO-QUBIT GENERALIZATION

For two qubits, $H_{2n+1}(\mathbb{Z}_2)$ corresponds to the n-qubit Pauli group. The mapping $(a,b,c) \mapsto i^c X^{a_1}Z^{b_1} \otimes X^{a_2}Z^{b_2}$ underlies stabilizer codes, Clifford groups, and quantum error correction.

Trigonometric Connection
Pauli matrices arise from the exponential form $e^{i\theta\sigma_k} = \cos\theta\, I + i \sin\theta\, \sigma_k$. At $\theta = \pi/2$, $e^{i\pi\sigma_x/2} = iX$ and $e^{i\pi\sigma_z/2} = iZ$. Hence the structure $N = (P_1, +, \cdot)$ is a finite near-ring.

VI. CONCLUSION

Finite trigonometric near-rings provide a unified algebraic framework linking trigonometry, non-commutative algebra, and quantum computation.

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