Unsteady Ternary Hybrid Nanofluid Flow with Variable Thermophysical Properties: A Physics-Informed Neural Network and Sensitivity Analysis Approach

FELIX JOHN FAWEHINMI[1], MOSES SUNDAY DADA[2]
[1, 2]*University of Ilorin*

Abstract- ***The investigation of ternary hybrid nanofluids (THNFs) has garnered significant attention due to their exceptional thermophysical properties and potential applications in advanced thermal management systems. This study presents a comprehensive numerical investigation of unsteady stagnation-point flow of a ternary hybrid nanofluid over a convectively heated sheet, incorporating temperature-dependent viscosity and thermal conductivity variations, magnetohydrodynamic (MHD) effects, Brownian motion, thermophoresis, viscous dissipation, and chemical reaction. The governing partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations. Physics-Informed Neural Networks (PINNs) are employed to obtain accurate numerical solutions, with the framework incorporating the governing equations, boundary conditions, and informed initial guesses into the loss function. The thermal performance of various ternary nanofluid compositions is evaluated, with MWCNTs/Al_2O_3/TiO_2 demonstrating the highest efficiency enhancement of 15.097%. A comprehensive parametric analysis reveals that the viscosity variation parameter (Λ) exerts the most dominant influence on momentum and transport characteristics, followed by unsteadiness (β) and thermal conductivity (ε). Response Surface Methodology (RSM) coupled with Central Composite Design (CCD) establishes quadratic regression models for skin friction, Nusselt, and Sherwood numbers, with ANOVA confirming statistical significance ($R^2 > 0.987$). Sensitivity analysis quantifies the relative impact of parameters, showing that Λ positively influences all transport quantities, while ε negatively affects skin friction and Nusselt number. This study demonstrates that PINNs provide a robust, mesh-free computational framework for complex nanofluid flows, offering superior accuracy and flexibility compared to traditional numerical methods.***

Keywords: Ternary Hybrid Nanofluid, Physics-Informed Neural Networks, Variable Viscosity, Variable Thermal Conductivity, Magnetohydrodynamics, Sensitivity Analysis, Response Surface Methodology

I. INTRODUCTION

The escalating demand for efficient thermal management systems in engineering applications, ranging from electronic cooling to solar energy systems and industrial heat exchangers, has driven significant research interest in advanced heat transfer fluids. Conventional heat transfer fluids such as water, ethylene glycol, and oils exhibit limited thermal conductivity, constraining their effectiveness in high-performance thermal systems (Jama et al., 2016). The advent of nanofluids: colloidal suspensions of nanometer-sized particles in base fluids represented a paradigm shift in heat transfer enhancement, with nanoparticles substantially increasing thermal conductivity while modifying rheological properties (Choi and Eastman, 1995).

The evolution from mono nanofluids to hybrid nanofluids (HNFs), comprising two distinct nanoparticle types dispersed in a base fluid, marked a significant advancement in thermal fluid engineering. Hybrid nanofluids exhibit superior thermophysical, optical, rheological, and morphological properties compared to their mono counterparts, demonstrating enhanced thermal conductivity, reduced pressure drop frictional losses, and improved stability (Shah et al., 2020; Rasheed et al., 2021). The synergistic effects of multiple nanoparticle species enable tailored property optimization for specific applications, including photovoltaic-thermal systems, electronic component cooling, and automotive thermal management (Adun et al., 2021; Suneetha et al., 2022).

Ternary hybrid nanofluids (THNFs) represents the most recent development in nanofluid technology which involve the integration of three distinct nanoparticle types within a single base fluid, offering unprecedented thermal performance enhancement. Studies have demonstrated that THNFs such as Al_2O_3-ZnO-Fe_3O_4 and Cu-Al_2O_3-Fe_3O_4 outperform both hybrid and mono nanofluids in energy and exergy efficiency, despite challenges associated with pressure drop and pumping power requirements (Adun et al., 2021; Raza et al., 2025). The incorporation of carbon-based nanoparticles such as MWCNTs further augments thermal and electrical properties, making THNFs particularly promising for energy-intensive and multiphysics systems (Abdelkader et al., 2024).

The flow behavior of nanofluids is significantly influenced by temperature-dependent variations in viscosity and thermal conductivity. These variable thermophysical properties introduce strong nonlinear coupling between momentum and energy transport mechanisms, substantially complicating the governing equations (Pearson, 1977; Ali et al., 2017). In microscale systems, accounting for temperature-variable properties can alter local Nusselt numbers by up to 30%, underscoring the necessity of incorporating variable property effects in accurate thermal modeling (Kakaç et al., 2011). The combined effects of viscosity and thermal conductivity variations, particularly in unsteady magnetohydrodynamic flows, remain insufficiently characterized, representing a critical gap in the literature.

The mathematical modeling of nanofluid flows typically yields systems of nonlinear partial differential equations that are analytically intractable. Traditional computational fluid dynamics (CFD) methods, while effective, face significant limitations including high computational costs, mesh generation difficulties in complex geometries, and challenges in handling high-dimensional, nonlinear, or chaotic systems (Wang et al., 2024). The emergence of Physics-Informed Neural Networks (PINNs) has introduced a transformative alternative by embedding governing physical laws directly into the neural network training process, combining the interpretability of physics-based models with the flexibility of machine learning (Raissi et al., 2019; Ghalambaz et al., 2024).

PINNs have demonstrated remarkable success in fluid dynamics applications, offering several distinct advantages over traditional approaches. The mesh-free nature of PINNs eliminates grid generation constraints, making them particularly suitable for complex geometries and high-dimensional systems (Grossmann et al., 2024). Moreover, PINNs naturally handle both forward and inverse problems without requiring large datasets, integrating partial differential equations, initial conditions, and boundary conditions into the loss function (Cai et al., 2021). Recent innovations including adaptive activation functions and curriculum learning have further enhanced PINN stability and extrapolation performance in fluid problems (Abbasi and Andersen, 2024; Le-Duc et al., 2024).

Sensitivity analysis has emerged as a crucial tool for understanding the relative importance of governing parameters in nanofluid systems. Studies employing Response Surface Methodology (RSM) and Artificial Neural Networks have elucidated the dominant roles of Brownian motion, nanoparticle volume fraction, and temperature in controlling heat transfer and rheological behavior (Mahanthesh et al., 2021; Sepehrnia et al., 2024; Islam et al., 2023). These analyses not only identify critical parameters but also guide the optimal design of nanofluid-based thermal systems.

Despite significant advances, the combined effects of variable viscosity and thermal conductivity in unsteady ternary hybrid nanofluid flows have not been investigated using physics-informed neural networks or other artificial intelligence-based approaches. The present study addresses this gap by employing PINNs coupled with sensitivity analysis to investigate the unsteady flow behavior of ternary hybrid nanofluids over a convectively heated sheet. The investigation incorporates variable viscosity and thermal conductivity, magnetic fields, Brownian motion, thermophoresis, viscous dissipation, and chemical reaction effects. The nanoparticles considered include copper (Cu), silver (Ag), multi-

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walled carbon nanotubes (MWCNTs), silicon carbide (SiC), and copper oxide (CuO), combined with aluminum oxide and titanium dioxide (Al_2O_3/TiO_2) to form various ternary compositions.

II. MATHEMATICAL FORMULATION

2.1 Governing Equations

The flow configuration consists of an unsteady, two-dimensional, incompressible ternary nanofluid over a vertical stretching plate, subject to an external magnetic and electric field. Temperature-dependent thermophysical properties are incorporated to capture the influence of variable viscosity and thermal conductivity, while additional transport mechanisms including viscous dissipation, Joule heating, heat generation/absorption, Brownian motion, thermophoresis, and chemical reaction are accounted for.

The following assumptions guide the formulation:

1. The flow is laminar, incompressible, and two-dimensional over a vertical stretching plate.
2. The base fluid is non-Newtonian, and nanoparticles (MWCNTs, Al_2O_3, TiO_2) are uniformly suspended, with no slip between phases.
3. Thermophysical properties including viscosity $\mu_{thnf}(T)$ and thermal conductivity $k_{thnf}(T)$ are temperature-dependent.
4. A uniform, time-dependent magnetic field $B_0(t)$ is applied normal to the plate, with the induced magnetic field neglected.
5. Heat transfer mechanisms include conduction, viscous dissipation, Joule heating, and internal heat generation/absorption.
6. Nanoparticle concentration dynamics are influenced by Brownian diffusion, thermophoresis, and a first-order chemical reaction.
7. Standard boundary layer approximations are invoked.

The governing equations for the ternary hybrid nanofluid flow are formulated by applying the fundamental principles of conservation of mass, momentum, energy, and species concentration. The continuity equation is given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

The Brinkman-extended Navier-Stokes equation incorporating electromagnetic body force yields:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial u_e}{\partial x}+u_e\frac{\partial u_e}{\partial x}+\frac{1}{\rho_{thnf}}\frac{\partial}{\partial y}\left(\mu_{thnf}(T)\frac{\partial u}{\partial y}\right)-\frac{\sigma_{thnf}}{\rho_{thnf}}B_0^2(t)(u-u_e)$$

The energy equation accounting for variable thermal conductivity, viscous dissipation, Joule heating, and heat generation is:

$$\begin{aligned}\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}
&=\frac{1}{(\rho c_p)_{thnf}}\left(k_{thnf}(T)+\frac{16\sigma^* T_\infty^3}{3K^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{thnf}(T)}{(\rho c_p)_{thnf}}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_{thnf}}{(\rho c_p)_{thnf}}B_0^2(u-u_e)^2\\
&+\frac{Q_0}{(\rho c_p)_{thnf}}(T-T_\infty)\end{aligned}$$

The concentration equation incorporating Brownian diffusion and thermophoresis is:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}$$

The boundary conditions are:

$$u = u_w(x,t), v = 0, T = T_w, C = C_w \text{ at } y = 0$$
$$u \to u_e(x,t), T \to T_\infty, C \to C_\infty \text{ as } y \to \infty$$

2.2 Similarity Transformations
The following similarity transformations are introduced to reduce the governing partial differential equations to ordinary differential equations:

$$\Psi = \left(\frac{a\nu}{1-ct}\right)^{1/2} x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \left(\frac{a}{\nu(1-ct)}\right)^{1/2} y, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where Ψ is the stream function satisfying u = ∂Ψ/∂y and v = -∂Ψ/∂x.

The wall temperature, concentration, and velocities are prescribed as:

$$T_w(x,t) = T_\infty + T_0\frac{ax^2}{2\nu}(1-ct)^{-3/2}, \qquad C_w(x,t) = C_\infty + C_0\frac{ax^2}{2\nu}(1-ct)^{-3/2},$$
$$u_w(x,t) = \frac{bx}{1-ct}, u_e(x,t) = \frac{ax}{1-ct}$$

The temperature-dependent viscosity and thermal conductivity are defined as:

$$\mu_{thnf}(T) = \mu_{thnf}(1 - \Lambda\theta(\eta)), k_{thnf}(T) = k_{thnf}(1 + \epsilon\theta(\eta))$$

2.3 Transformed Equations
Upon applying the similarity transformations, the governing equations reduce to the following system of coupled nonlinear ordinary differential equations:

$$(1-\Lambda\theta)f''' + \alpha_1 M(f'-1) - \alpha_2\left[\beta\left(\frac{\eta}{2}f'' + f' + 1\right) + (f')^2 - ff'' + 1\right] = 0,$$
$$(1 + \epsilon\theta + R_d)\theta'' + \epsilon(1+\epsilon\theta)\theta'^2$$
$$+ Pr\left[\alpha_3(1-\Lambda\theta)Ec(f'')^2 + \alpha_4\left(f\theta' - 2f'\theta + Q\theta - \frac{\beta}{2}(3\theta + \eta\theta')\right)\right.$$
$$\left. + \alpha_5 MEc(f'-1)^2\right] = 0, \qquad \phi'' + \frac{N_t}{N_b}\theta'' + PrLe\left[f\theta' - \frac{\beta}{2}(3\phi + \eta\phi') - f''\phi\right] = 0$$

The transformed boundary conditions are:

$$f(0) = 0, f'(0) = \alpha, \theta(0) = 1, \phi(0) = 1,$$
$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0$$

The dimensionless parameters are defined as:
M = magnetic parameter, β = unsteadiness ratio parameter, α = stretching ratio parameter, Pr = Prandtl number, Ec = Eckert number, Q = heat generation/absorption parameter, Le = Lewis number, Rd = thermal radiation parameter, Nt = thermophoresis parameter, Nb = Brownian motion parameter, Λ =viscosity variation parameter, ε: thermal conductivity variation parameter.

The thermophysical property ratios are defined as:

$$\alpha_1 = \frac{\rho_{thnf}}{\rho_f}, \alpha_2 = \frac{\sigma_{thnf}}{\mu_f}, \alpha_3 = \frac{\mu_{thnf}}{k_{thnf}/k_f}, \alpha_4 = \frac{\rho_{thnf}}{k_{thnf}/k_f}, \alpha_5 = \frac{\sigma_{thnf}}{k_{thnf}/k_f}$$

2.4 Engineering Quantities
The skin friction coefficient (Cf), local Nusselt number (Nux), and Sherwood number (Sh) are defined as:

$$\sqrt{Re_x} C_f = \frac{\mu_{thnf}}{\mu_f}(1-\Lambda) f''(0) \frac{Nu_x}{\sqrt{Re_x}} = -\frac{k_{thnf}(1+\epsilon)}{k_f}\theta'(0) \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0)$$

where Re_x is the local Reynolds number.

III. PHYSICS-INFORMED NEURAL NETWORK METHODOLOGY

3.1 Fundamental Principles
Physics-Informed Neural Networks (PINNs), originally proposed by Raissi et al. (2019), extend classical neural networks by embedding physical constraints into the training process. Instead of relying solely on data, PINNs incorporate the governing partial differential equations, initial conditions, and boundary conditions directly into the loss function.

Consider a general nonlinear PDE of the form:

$$u_t + \mathcal{N}[u] = 0, x \in \Omega, t \in [0, T]$$

where $\mathcal{N}$ is a nonlinear differential operator and u(t,x) is the latent solution of interest. The associated initial and boundary conditions are:

$$u(0, x) = I_0(x), x \in \Omega u(t, x) = B_d(t, x), x \in \partial\Omega, t \in [0, T]$$

Define f(t,x) as the residual of the PDE:

$$f(t, x) \coloneqq u_t + \mathcal{N}[u]$$

The PINN framework learns u(t,x) by training the neural network to minimize the mean squared error (MSE) loss:

$$\mathrm{MSE} = \mathrm{MSE}_f + \mathrm{MSE}_i + \mathrm{MSE}_b$$

where:

$$\mathrm{MSE}_f = \frac{1}{N_f}\sum_{i=1}^{N_f} |f_\lambda(t_i^f, x_i^f)|^2 = \frac{1}{N_f}\sum_{i=1}^{N_f} |u_t(t_i^f, x_i^f) + \mathcal{N}[u_\lambda](t_i^f, x_i^f)|^2$$

$$\mathrm{MSE}_i = \frac{1}{N_u}\sum_{i=1}^{N_u} |u_\lambda(0, x_i^u) - I_0(x_i^u)|^2 \quad \mathrm{MSE}_b = \frac{1}{N_b}\sum_{i=1}^{N_b} |u_\lambda(t_i^u, x_i^u) - B_d(t_i^u, x_i^u)|^2$$

The solution u(t,x) is approximated using a deep feed-forward neural network parameterized by λ, with automatic differentiation used to compute required derivatives for PDE residuals.

3.2 PINN Implementation for Ternary Hybrid Nanofluid Flow

A feed-forward network with hyperbolic tangent activations and a small explicit pretraining phase was employed to align the network with physically-motivated initial guesses for velocity, temperature, and concentration distributions.

The initial guesses are defined as:

$$f_0(\eta) = \eta - (1-\alpha)e^{-\eta}, \theta_0(\eta) = e^{-\eta}, \phi_0(\eta) = e^{-\eta}$$

which satisfy the boundary conditions:

$$f_0(0) = 0, f_0'(0) = \alpha, \theta_0(0) = 1, \phi_0(0) = 1 f_0'(\infty) = 1, \theta_0(\infty) = 0, \phi_0(\infty) = 0$$

For pretraining, 500 epochs were used to minimize the mean-squared deviation:

$$\mathcal{L}_{pre} = \frac{1}{N_c}\sum_{i=1}^{N_c} |u_{NN}(\eta_i) - u_{guess}(\eta_i)|^2$$

providing a well-conditioned starting point for optimization. Subsequently, PINN training was performed with 4500 epochs using the Adam optimizer to minimize the total composite loss.

The residuals of the governing differential equations are defined as:

$$R_f(\eta) \coloneqq (1-\Lambda\theta)f''' + \alpha_1 M(f'-1) - \alpha_2\left[\beta\left(\frac{\eta}{2}f''+f'+1\right) + (f')^2 - ff'' + 1\right]$$

$$\begin{aligned} R_\theta(\eta) \coloneqq{}& (1+\epsilon\theta+R_d)\theta'' + \epsilon(1+\epsilon\theta)\theta'^2 \\ &+ Pr\left[\alpha_3(1-\Lambda\theta)Ec(f'')^2 + \alpha_4\left(f\theta' - 2f'\theta + Q\theta - \frac{\beta}{2}(3\theta+\eta\theta')\right) + \alpha_5 MEc(f' \right. \\ &\left. - 1)^2\right] \end{aligned}$$

$$R_\phi(\eta) := \phi'' + \frac{N_t}{N_b}\theta'' + PrLe\left[f\theta' - \frac{\beta}{2}(3\phi + \eta\phi') - f''\phi\right]$$

3.3 Sensitivity Analysis and Response Surface Methodology

The Response Surface Methodology (RSM) and Central Composite Design (CCD) were employed to analyze the fundamental engineering quantities. A quadratic response surface model of the form:

$$y = \beta_0 + \sum_{i=1}^{3} \beta_i x_i + \sum_{i=1}^{3} \beta_{ii} x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j + \epsilon$$

was constructed, where the independent variables Λ, ε, and β were investigated at three levels (-1, 0, 1).
Sensitivity indices were computed using partial derivatives of the response functions:

$$\frac{\partial C_f}{\partial \Lambda} = 0.286137 + 1.034790\Lambda + 0.077129\epsilon - 0.031102\beta$$
$$\frac{\partial C_f}{\partial \epsilon} = -0.381519 - 0.017096\epsilon + 0.077129\Lambda + 0.005022\beta$$
$$\frac{\partial Nu}{\partial \Lambda} = 0.326763 + 0.597186\Lambda - 0.002243\epsilon + 0.118592\beta$$
$$\frac{\partial Nu}{\partial \epsilon} = -0.072842 - 0.040966\epsilon - 0.002243\Lambda - 0.003547\beta$$
$$\frac{\partial Sh}{\partial \Lambda} = 0.496393 - 0.021018\Lambda - 0.034203\epsilon - 0.036813\beta$$
$$\frac{\partial Sh}{\partial \epsilon} = 0.027908 - 0.085670\epsilon - 0.034203\Lambda - 0.045317\beta$$

3.4 Response Surface Methodology and ANOVA Analysis

The Central Composite Design (CCD) matrix in Table 4.5 encapsulates the numerical results from PINN simulations for the ternary hybrid nanofluid system. The three independent dimensionless parameters—viscosity variation (Λ), thermal conductivity (ε), and unsteadiness (β)—were varied systematically to evaluate their effects on skin friction coefficient (Cf), local Nusselt number (Nux), and local Sherwood number (Sh).

The quadratic regression models for the three responses are:

$$C_f = 1.453769 + 0.286137\Lambda - 0.381519\epsilon - 0.085482\beta + 0.517395\Lambda^2 - 0.008548\epsilon^2 - 0.054300\beta^2 + 0.077129\Lambda\epsilon - 0.031102\Lambda\beta + 0.005022\epsilon\beta$$

$$Nu = 2.633203 + 0.326763\Lambda - 0.072842\epsilon + 0.086366\beta + 0.298593\Lambda^2 - 0.020483\epsilon^2 - 0.080467\beta^2 - 0.002243\Lambda\epsilon + 0.118592\Lambda\beta - 0.003547\epsilon\beta$$

$$Sh = 2.063238 + 0.496393\Lambda + 0.027908\epsilon - 0.009819\beta - 0.010509\Lambda^2 - 0.042835\epsilon^2 - 0.057263\beta^2 - 0.034203\Lambda\epsilon - 0.036813\Lambda\beta - 0.045317\epsilon\beta$$

The ANOVA analysis (Tables 4.7-4.9) reveals that Λ and ε play dominant roles in influencing momentum and thermal transport with highly significant effects ($p < 0.001$), whereas Λ alone substantially affects mass transfer. The model accuracies are exceptional with R^2 values of 0.989, 0.992, and 0.987 for Cf, Nu, and Sh respectively, with adjusted R^2 values of 0.978, 0.984, and 0.975, confirming the adequacy and robustness of the quadratic response surface models.

3.5 Sensitivity Analysis

The sensitivity indices presented quantify the impact of parameters on transport phenomena. The partial derivatives reveal that:

- $\partial Cf/\partial\Lambda$ is positive across all levels, indicating that viscosity variation enhances skin friction through momentum boundary layer thickening.
- $\partial Cf/\partial\varepsilon$ remains negative, showing that thermal conductivity reduces wall shear through enhanced diffusion.
- $\partial Nu/\partial\Lambda$ and $\partial Nu/\partial\beta$ are positive, implying stronger heat transfer under viscous and unsteady influences.
- $\partial Nu/\partial\varepsilon$ is negative, reflecting reduced wall heat flux with increased thermal conductivity.
- $\partial Sh/\partial\Lambda$ and $\partial Sh/\partial\varepsilon$ are generally positive at low β but decrease as β increases.
- $\partial Sh/\partial\beta$ becomes negative, showing that higher unsteadiness suppresses mass transfer.

IV. DISCUSSION AND CONCLUSION

4.1 Discussion

Experimental results reveal a clear hierarchy of performance related to the heating process, where ternary hybrid nanofluids have proven better than mono and hybrid nanofluids. Among all investigated compositions, the combination of MWCNTs/Al2O3/TiO2 showed the highest efficiency equal to 15.097%. It can be explained by the synergetic effect of mixing nanostructures based on carbon with oxide nanoparticles. Such an increase in performance is due to high thermal conductivity characteristics of MWCNTs, which allow heat to be conducted quickly, along with dispersion stability and improved interfacial heat transfer of oxides. An increase in efficiency from Cu (5.74%) to Cu/Al2O3/TiO2 (15.08%) and from SiC (5.67%) to SiC/Al2O3/TiO2 (15.01%) proves the amplification of the effects of each component through ternary combinations.

The velocity field shows that there is a perfect balance between the acting driving forces and resisting forces which act on the flow of fluids. Factors such as unsteady parameter, Eckert number, and thermal conductivity favor the increase in velocity by lowering the viscosity of fluids or enhancing the process of thermal diffusion while parameters such as magnetic parameter and viscosity reduce the fluid flow through Lorentz force and higher momentum resistance respectively. The same applies for Brownian motion and thermophoresis which have contrasting effects on flow; while Brownian motion favors increased viscosity leading to slower flow, thermophoresis affects density differences which results in lower acceleration. The temperature profile reacts conversely to factors such as viscous dissipation and heat generation which contribute towards heating while convection and magnetic field favor cooling effect. It is noteworthy that the thermal conductivity and Prandtl number increase temperature levels differently.

The concentration field also displays a similar pattern of boundary layer flow due to the combined effect of convection, diffusion, and chemical reactions. High fluid velocities and strong magnetic field have an effect of decreasing the accumulation of the solute near the wall while viscosity, heat mass interaction, and low diffusivity lead to increased concentrations. Thermophoresis continues playing its dispersing role by taking away the particles from the surface thus resulting in a decrease in concentration. The impact of the reaction parameter on all three fields in the form of increasing both velocity, temperature, and concentration shows the importance of chemical reaction in terms of releasing energy and increasing the species generation rate.

4.2 Conclusion

This comprehensive investigation of unsteady ternary hybrid nanofluid flow with variable thermophysical properties using Physics-Informed Neural Networks

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and sensitivity analysis yields the following key conclusions:

1. Thermal Performance Enhancement: Ternary hybrid nanofluids demonstrate significant thermal performance enhancement, with MWCNTs/Al_2O_3/TiO_2 achieving the highest efficiency of 15.097%, confirming the synergistic effects of multi-component nanoparticle suspensions.
2. Parameter Dominance: The viscosity variation parameter (Λ) exerts the most dominant influence on momentum and transport behavior, followed by unsteadiness (β) and thermal conductivity (ε). This establishes the critical importance of accurate viscosity modeling in nanofluid applications.
3. Transport Mechanisms: Parameters promoting fluid motion (α, β, Ec, ε) generally enhance velocity and reduce temperature, while resistive parameters (M, Rd, Λ) suppress velocity and elevate temperature. The combined analysis highlights the complex interplay between electromagnetic, thermal, and rheological mechanisms.
4. Statistical Validation: The quadratic regression models demonstrate exceptional predictive capability with R^2 values exceeding 0.987, and ANOVA confirms the statistical significance of the developed models ($p < 0.001$), validating the robustness of the parametric analysis and optimization framework.
5. Sensitivity Patterns: Viscosity variation and unsteadiness generally enhance transport phenomena, whereas thermal conductivity weakens them. Specifically, Λ positively influences skin friction, Nusselt, and Sherwood numbers, while ε negatively influences Cf and Nu.
6. PINN Efficacy: Physics-Informed Neural Networks provide a robust, mesh-free computational framework for complex nanofluid flows, accurately capturing nonlinear dynamics while maintaining physical consistency across the computational domain.
7. Engineering Significance: The established predictive and optimization framework using RSM, sensitivity analysis, and machine learning provides a foundation for designing high-performance thermal systems employing ternary hybrid nanofluids.

The findings demonstrate that ternary hybrid nanofluids, particularly those incorporating carbon-based nanoparticles, represent a promising pathway for next-generation thermal management systems. The integration of PINNs with sensitivity analysis offers a powerful, efficient methodology for investigating complex nanofluid flows, with potential applications extending to other ultiphysics transport phenomena.

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