

Heat Transfer Analysis of Non-Newtonian Nanofluids Across a Horizontal Porous Surface Under Chemical Reaction

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Abstract- *This work investigates the heat transfer behavior of ordinary Williamson and Williamson nanofluid flows over a horizontal porous surface under the influence of magnetic fields, solar radiation, binary chemical reactions, and activation energy. A mathematical model incorporating thermophoresis and Brownian motion is developed, and the governing boundary value problems are solved numerically using the shooting method. Comparative analysis of temperature profiles and heat transfer rates is presented through graphs and tables. The results demonstrate that Williamson nanofluids exhibit superior thermal performance owing to the inclusion of nanoparticles, underscoring their potential for diverse industrial applications. This study investigates the influence of cross-diffusion phenomena on the three-dimensional flow and heat transfer characteristics of a Jeffrey nanofluid confined within a Lorentz force driven stretchable channel. The rheological behavior of the non-Newtonian Jeffrey fluid model is incorporated to account for relaxation and retardation effects, while the impact of an applied magnetic field introduces magnetohydrodynamic (MHD) forces into the system. The thermal analysis considers nonlinear radiation to capture high-temperature effects, and cross-diffusion mechanisms are modeled through the Soret and Dufour effects to describe coupled heat and mass transport. The resulting boundary value problem is solved numerically to examine the effects of key physical parameters on velocity, temperature, and concentration distributions. The cross-diffusion analysis demonstrates that an increase in the Dufour number significantly augments the temperature and velocity profiles due to energy flux induced by concentration gradients, while simultaneously diminishing the concentration distribution. In contrast, elevated values of the Soret number enhance both the concentration and velocity fields by intensifying mass diffusion driven by thermal gradients within the boundary layer.*

I. INTRODUCTION

In modern research, understanding the mechanisms of non-Newtonian materials has become a focal point. The rheological characteristics of non-Newtonian materials significantly differ from those of Newtonian materials. Various substances such as shampoos, toothpaste, soaps, honey, sugar solutions, polymers, blood, ketchup, applesauce, drilling muds, and lubricants fall under the category of non-Newtonian fluids. All these liquids exhibit various rheological properties and mechanisms. In non-Newtonian fluids, pseudoplastic fluids stand out as among the most commonly encountered types. The Williamson fluid represents a pseudoplastic type of non-Newtonian fluid characterized by shear-thinning behavior. The Williamson fluid holds significant importance due to its applications in various fields including lubricants, biomedical fluids, emulsions, and nuclear fuel slurries. Williamson [1] explained the flow behavior of pseudoplastic materials and introduced a model to describe their characteristic flow. Experimental validation confirmed, the viscosity decrease with increasing rate of shear stress. Lyubimov and Perminov [2] investigated the effects of gravitational force on a thin layer of Williamson fluid. Nadeem et al. [3] formulated a model where they considered chyme as a Williamson fluid, and they conducted the flow analysis within the annular region created by two concentric tubes. Vajravelu and Dhivya [4] conducted a numerical analysis of Williamson fluid flow over a moving vertical cylinder with variable porosity using the Crank-Nicholson method. Salawu [5] numerically analyzed the stagnation-point flow with considering the influence of activation energy within a Williamson fluid comprising tiny particles over an expansive plate.

The present study investigates the MHD flow of a Williamson nanofluid across an exponential stretching surface, taking into account nonlinear thermal radiation to enhance heat transfer. While stretching scenarios have been extensively analyzed with linear thermal radiation, there has been limited attention given to flow situations involving nonlinear thermal radiation. However, the impact of thermal radiation plays a crucial role in influencing the heat transfer rate and temperature distributions within the boundary layer flow of the participating fluid. This phenomenon finds various practical applications in industries such as metallurgy, manufacturing, and energy generation. Additionally, it holds significance in electronics by impacting the performance and cooling of electronic components. Notably, one of the most notable practical uses of this concept is in utilizing solar radiation as an energy source on Earth. The study conducted by Rashidi et al. [6] examined the MHD stretched flow of a nanofluid under the influence of buoyancy and thermal radiation. A salient feature of thermal radiation in nanofluid unsteady flow over a stretching sheet was reported by Das et al. [7], the study examined the impact of thermal radiation in a time-dependent magnetohydrodynamic flow with varying viscosity. Mahanthesh et al. [8] conducted an investigation into the radiative flow of a hydromagnetic nano-fluid over the rotation of a disk. Their analysis included a nanofluid composed of water-based nanoparticles, considering various shapes of nanoparticles such as lamina, column, sphere, tetrahedron, and hexahedron. Khan et al [9] explored a mathematical model for entropy generation incorporating variable fluid properties. They further examined the impact of mixed convection and nonlinear radiation. Reddy et al. [10] conducted a comprehensive examination of radiative heat transfer in Casson nanofluid, while also considering the viscous dissipation and particle movement. Moreover, recent research efforts have expanded beyond examining individual components to provide a complete understanding of thermal radiation's influence on various flows [11-18]. These studies provide a comprehensive understanding of the various impacts of thermal radiation in diverse fluid dynamics situations.

The motion of Williamson liquid combined with chemical reactions has found extensive applications in various fields such as drying processes, geothermal reservoirs, surface dehydration, enhanced oil recovery, geothermal pools, fibrous insulation, food processing, thermal insulation, iron rusting, fog formation, nuclear reactor cooling, synthetic materials, and numerous others. Due to extensive applications, the consequence of chemical reaction has been investigated and reported by many researchers Mukhopadhyay et al [19], Hayat et al. [20], Umavathi et al. [21], Mustafa et al. [22], Majeed et al. [23] and Nadeem et al [24].

Common liquids are often underutilized in various scientific and technical sectors due to their poor heat conductivity. However, nanoparticles are gaining significant attention in this era due to their remarkable thermal impact and unique applications across industries, biological, and engineering sectors. Such as nuclear power, paper production, insulation collectors, glass-fibre manufacture, geothermal energy pipe cooling systems, and heat transmission in aircraft apparatus. Nanoparticles, characterized by their small metallic particles ranging from 1 to 100 nm, possess enhanced thermo-physical properties. Recent studies have highlighted that nanofluids exhibit higher thermal conductivity compared to conventional fluids. Choi [25] provided experimental validation of nanofluids, establishing the existence of this concept. Throughout the last two decades, researchers around the world have been fascinated by the exceptional properties of nanofluid, remarkable studies being recorded in the field [26-34].

The flow and heat transfer characteristics of non-Newtonian fluids, such as Williamson fluids, are of significant interest due to their complex rheological behavior. When these fluids are enhanced with nanoparticles, forming nanofluids, their thermal properties are further improved. This study focuses on the comparative analysis of Williamson and Williamson nanofluid flows across a horizontal porous surface in the presence of magnetic fields, solar radiation, binary chemical reactions and activation energy. The presenting governing equations are highly nonlinear mathematical expressions that are numerically treated to examine

the outcomes. Graphical representations are employed and discussed to express the mechanisms underlying various physical constraints on dimensionless quantities. The numerical computations cover two distinct scenarios: Williamson nanofluid and ordinary Williamson fluid. The results reveal a notably higher heat transfer rate for the Williamson nanofluid in comparison to the ordinary Williamson fluid.

Mathematical Formulation

With all above assumptions the governing boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{k'} u + g(\beta_0(T - T_\infty) + \beta_1(T - T_\infty)^2 + \beta_2(C - C_\infty)) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[D_m \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q_0}{\rho c_p} (T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^m \exp \left(-\frac{E_a}{\kappa T} \right), \quad (4)$$

The radiative heat flux expression in equation (3) is given by the Rosseland approximation as;

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y} \quad (5)$$

Where σ^* and k^* are the Stefan-Boltzman constant and the mean absorption coefficient correspondingly, and in view to equation (5) in equation (3) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau \left[D_m \left(\frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{16\sigma^*}{3\rho c_p k^*} \left[T^3 \frac{\partial^2 T}{\partial y^2} + 3T^2 \left(\frac{\partial T}{\partial y} \right)^2 \right] \quad (6)$$

The boundary conditions considered for the present flow analysis are;

We consider a steady two-dimensional flow of Williamson fluid containing nanoparticles over an exponentially stretching surface. The flow configuration with the adopted coordinate system is shown in Figure 1, where the x-axis is taken along the stretching surface in the direction of motion and the y-axis is normal to it. The surface is exponentially stretched in the x-direction with a velocity $U_w(x) = U_0 e^{x/L}$. A magnetic field strength B_0 is applied along the y-direction. At the surface, the temperature and nanoparticle volume fraction of the nanofluid are denoted as T_w and C_w , respectively.

$$u = U_w(x) = U_0 e^{x/L}, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0, \\ u = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty. \quad (7)$$

The term $k_r^2(C - C_\infty) \left(\frac{T}{T_\infty}\right)^m \exp\left(-\frac{E_a}{\kappa T}\right)$ in equation (4) represents the modified Arrhenius equation in which k_r^2 is the reaction rate, E_a the activation energy, $\kappa = 8.61 \times 10^{-5} \text{ eV/K}$ the Boltzmann constant and m the fitted rate constant which generally lies in the range $-1 < m < 1$. where u and v are velocity components along x and y directions respectively, T and C are temperature and volume fraction of nanoparticles correspondingly, ν -kinematic viscosity, ρ -is density of the fluid, Q_0 -dimensional heat source coefficient, B_0 -magnetic field, k' -permeability of the porous medium, $\alpha_m = k/\rho c_p$ -thermal diffusivity of the fluid, k -thermal conductivity of the fluid, D_T -thermophoretic diffusion coefficient, D_m -Brownian diffusion coefficient, $\tau = (\rho c)_p/(\rho c)_f$ -ratio of the effective heat capacity of the nanoparticle to that of an ordinary fluid, $(\rho c)_f$ and $(\rho c)_p$ are heat capacities of the ordinary fluid and nanoparticles respectively, T_∞ and C_∞ are ambient temperature and volume fraction of nanoparticles respectively and q_r radiative heat flux.

Now, introduce the following similarity transformations

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \quad v = -\sqrt{\frac{U_0 \nu}{2L}} e^{\frac{x}{2L}} (f(\eta) + \eta f'(\eta)), \quad \eta = \sqrt{\frac{U_0}{2\nu L}} e^{x/2L} y \\ T = \theta(\eta)(T_w - T_\infty) + T_\infty, \quad C = \phi(\eta)(C_w - C_\infty) + C_\infty \quad (8)$$

With the help of above transformations, equation Eq. (1) is identically satisfied and Equation's. (2), (4) and (6) along with boundary conditions (7) take the following equations;

$$f''' + f f'' - 2(f')^2 + \omega f'' f''' - (M + kp)f' + \gamma(1 + \gamma_1)(\theta + \alpha\theta^2 + N\phi) = 0, \quad (9)$$

$$\theta'' + R \left[(1 + (\theta_w - 1)\theta(\eta))^3 \theta''(\eta) + 3(\theta_w - 1)\theta'^2(\eta)(1 + (\theta_w - 1)\theta(\eta))^2 \right] \\ \text{Pr}\{f\theta' + Nb\phi'\theta' + Nt(\theta')^2 + Q\theta\} = 0 \quad (10)$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' - Le\sigma(1 + \delta\theta)^m \phi \exp\left(-\frac{E}{1 + \delta\theta}\right) = 0, \quad (11)$$

and the corresponding boundary conditions become;

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1, \\ f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (12)$$

where $\omega = \Gamma \sqrt{\frac{U_0^2 e^{3x/L}}{\nu L}}$ -Williamson fluid Parameter, $M = \frac{2L\sigma B_0^2}{\rho U_w}$ -magnetic field parameter, $kp = \frac{2Lv}{k'U_w}$ - porous parameter $Pr = \frac{\nu}{\alpha_m}$ -Prandtl number, $R = \frac{16\sigma^* T_\infty^3}{3kk^*}$ -radiation parameter, $\theta_w = \frac{T_w}{T_\infty}$ -temperature parameter, $Nb = \frac{D_m(C_w - C_\infty)}{\nu}$ -Brownian motion parameter, $Nt = \frac{\tau D_T(T_w - T_\infty)}{T_\infty \nu}$ -thermophoresis parameter, $Le = \frac{\nu}{D_m}$ -Lewis number and $Q = \frac{Q_0 2L}{\rho c_p U_w}$ -heat generation parameter, $\sigma = -\frac{k_f^2 2L}{U_w}$ -dimensionless reaction rate, $\delta = \frac{T_w - T_\infty}{T_\infty}$ - the temperature difference parameter, $E = \frac{E_a}{\kappa T_\infty}$ -non-dimensional activation energy.

The Skin friction coefficient, Nusselt number and Sherwood numbers are;

$$C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (13)$$

Where τ_w -surface shear stress, q_w -surface heat flux and q_m -surface mass flux are given by;

$$\begin{aligned} \tau_w &= \mu \left(\frac{\partial u}{\partial y} + \frac{\Gamma}{2} \left(\frac{\partial u}{\partial y} \right)^2 \right)_{y=0} \\ q_w &= -k \left(\frac{\partial T}{\partial y} + q_r \right)_{y=0}, \\ q_m &= -D_m \left(\frac{\partial c}{\partial y} \right)_{y=0} \end{aligned} \quad (14)$$

Now through merging equation (8) and (14) in interpretation of equation (13), we have obtained;

$$\begin{aligned} \sqrt{Re_x} C_f &= \left(f''(0) + \frac{\lambda}{2} f''(0)^2 \right), \\ \frac{Nu_x}{\sqrt{2Re}} &= -\frac{x}{2L} (1 + R\theta_w^3) \theta'(0), \\ \frac{Sh_x}{\sqrt{2Re}} &= -\frac{x}{2L} \phi'(0). \end{aligned} \quad (15)$$

Where $Re_x = \frac{U_0 L}{\nu}$ is the local Reynolds number.

II. PHYSICAL INTERPRETATION

In this section, a comprehensive numerical investigation is carried out to examine the influence of the governing physical parameters on the flow, thermal, and nanoparticle transport characteristics. Particular attention is given to the velocity $f(\eta)$, temperature $\theta(\eta)$, and nanoparticle volume fraction

$\phi(\eta)$ profiles. The numerical results are presented through graphical illustrations and tables, and the corresponding physical mechanisms are discussed in detail. Unless otherwise stated, the remaining governing parameters are maintained at their prescribed default values, which are specified in the respective figures. Figures 2 and 3 illustrate the influence of the magnetic parameter M on the

velocity and temperature distributions, respectively. As M increases from 0.5 to 2, a clear reduction in the velocity profile is observed, accompanied by an increase in the temperature profile. The decrease in velocity is attributed to the enhanced Lorentz force generated by the interaction between the applied magnetic field and the electrically conducting fluid. This resistive electromagnetic force acts against the fluid motion and consequently suppresses the momentum transport, resulting in a thinner momentum boundary layer. On the other hand, the reduction in fluid motion weakens the convective transport of thermal energy away from the surface. Consequently, heat accumulates within the boundary layer, leading to an increase in the temperature and a corresponding thickening of the thermal boundary layer. Figure 4 presents the effect of the permeability parameter on the velocity distribution $f(\eta)$. It is evident that increasing the permeability parameter causes a reduction in the fluid velocity. The porous medium offers additional resistance to the fluid motion, thereby restricting the movement of the fluid through the porous structure. As the resistance offered by the porous matrix becomes stronger, the fluid motion is increasingly suppressed, resulting in a reduction in the velocity profile and momentum transport within the boundary layer. The corresponding influence of the permeability parameter on the temperature profile $\theta(\eta)$ is displayed in Figure 5. An enhancement in both the temperature profile and thermal boundary-layer thickness is observed with increasing permeability parameter. The increased resistance to fluid motion reduces the convective transport of heat from the surface into the surrounding fluid. As a result, the removal of thermal energy becomes less effective, causing heat to be retained near the surface and consequently increasing the temperature distribution. Figures 6 and 7 depict the effects of the Brownian motion parameter Nb on the temperature and nanoparticle volume-fraction profiles, respectively. An increase in Nb produces a noticeable enhancement in the temperature profile and thermal boundary-layer thickness. This behavior can be attributed to the intensified random motion of nanoparticles, which increases microscopic interactions between the nanoparticles and the base fluid and thereby modifies the thermal transport mechanism. In contrast, the nanoparticle volume-

fraction profile decreases as Nb increases, as observed in Figure 7. The stronger Brownian diffusion promotes the redistribution of nanoparticles away from the surface region, resulting in a reduction in the local nanoparticle concentration within the boundary layer. Thus, Brownian motion plays an important role in coupling nanoparticle diffusion with heat transport.

Figures 8 and 9 demonstrate the effects of the thermophoresis parameter Nt on the temperature and nanoparticle volume-fraction distributions, respectively. Increasing Nt leads to an enhancement in both temperature and nanoparticle volume-fraction profiles. The thermophoretic effect induces nanoparticle migration in response to the temperature gradient, generally driving particles from regions of higher temperature toward regions of lower temperature. This migration modifies the nanoparticle distribution and enhances the interaction between nanoparticle transport and thermal energy transfer. Consequently, the thermal and nanoparticle boundary layers become more pronounced as the thermophoresis parameter increases. Figure 10 illustrates the effect of the thermal radiation parameter R on the temperature distribution. It is observed that increasing R results in higher temperature values and a thicker thermal boundary layer. The presence of thermal radiation provides an additional mode of energy transport within the fluid, thereby increasing the overall thermal energy available in the boundary layer. Consequently, the temperature of the fluid increases and the thermal boundary layer becomes thicker. This result highlights the significance of radiative heat transfer in high-temperature thermal transport applications. Finally, Figure 11 presents the influence of the Lewis number Le on the nanoparticle volume-fraction profile. The nanoparticle concentration decreases progressively with increasing Le . Since the Lewis number represents the relative importance of thermal diffusivity to mass or nanoparticle diffusivity, a larger Le corresponds to weaker nanoparticle mass diffusion relative to thermal diffusion. Consequently, nanoparticle transport becomes less effective, leading to a reduction in the nanoparticle volume-fraction distribution and a thinner concentration boundary layer. This behavior confirms the important role of

the Lewis number in controlling nanoparticle boundary layer.
diffusion and concentration transport within the

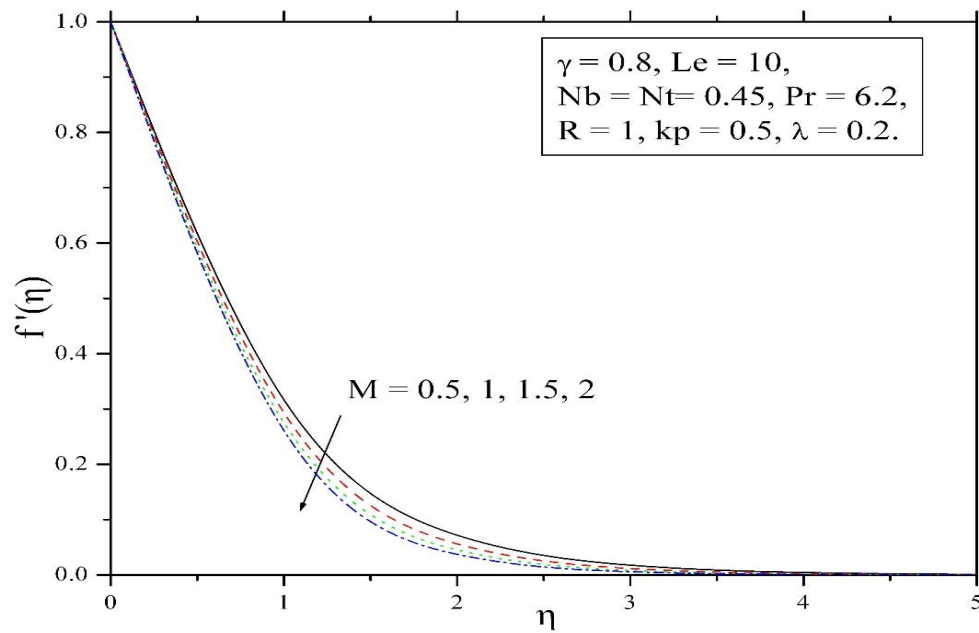


Figure 1: Effect of M on $f'(\eta)$

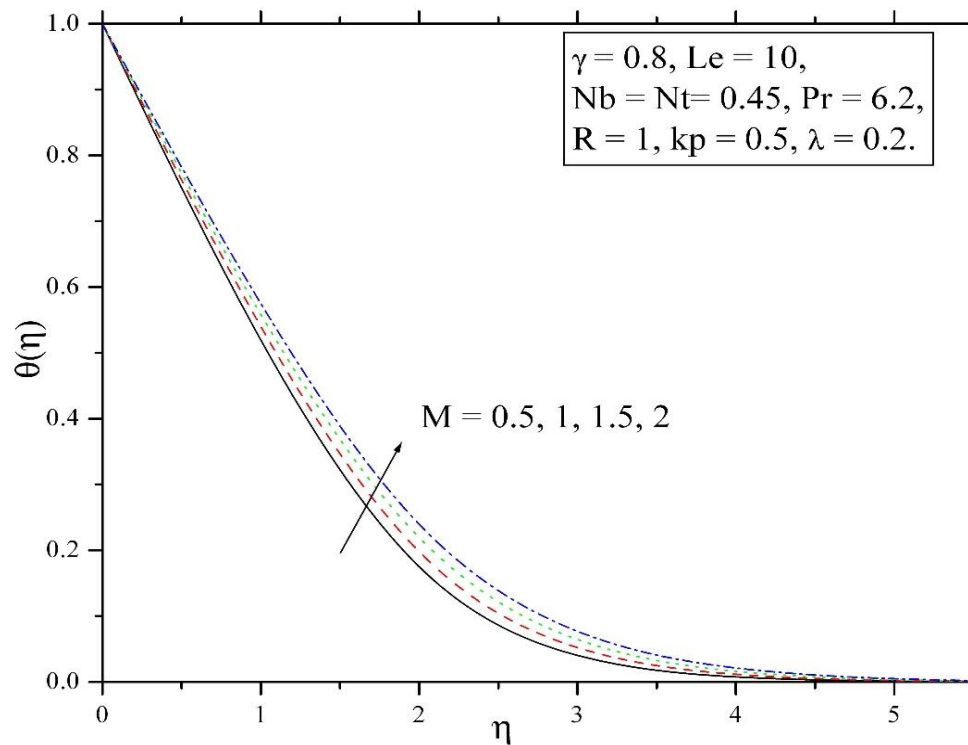


Figure 2: Effect of M on $\theta(\eta)$

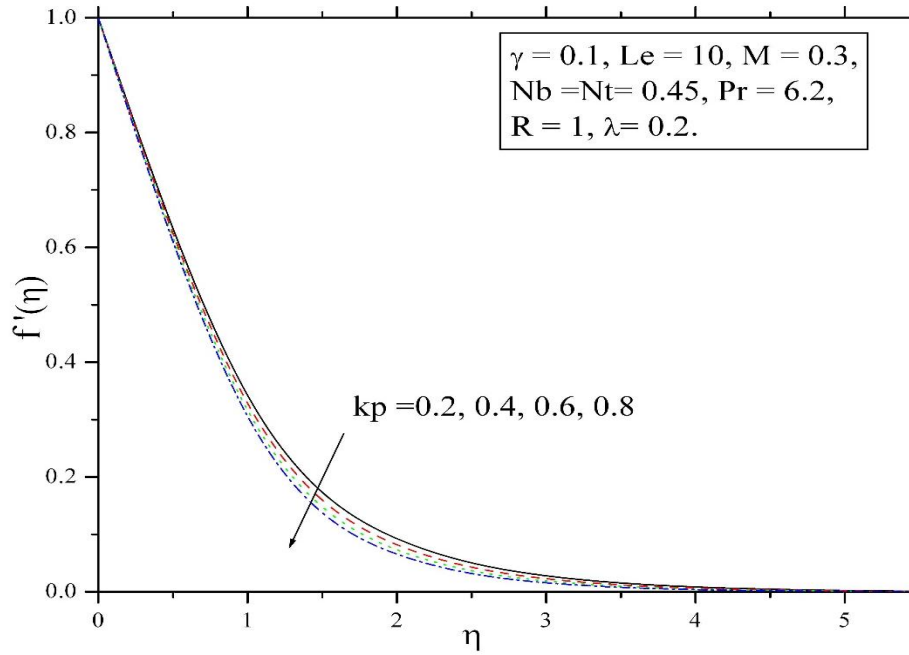


Figure 3: Effect of K_p on $f'(\eta)$.

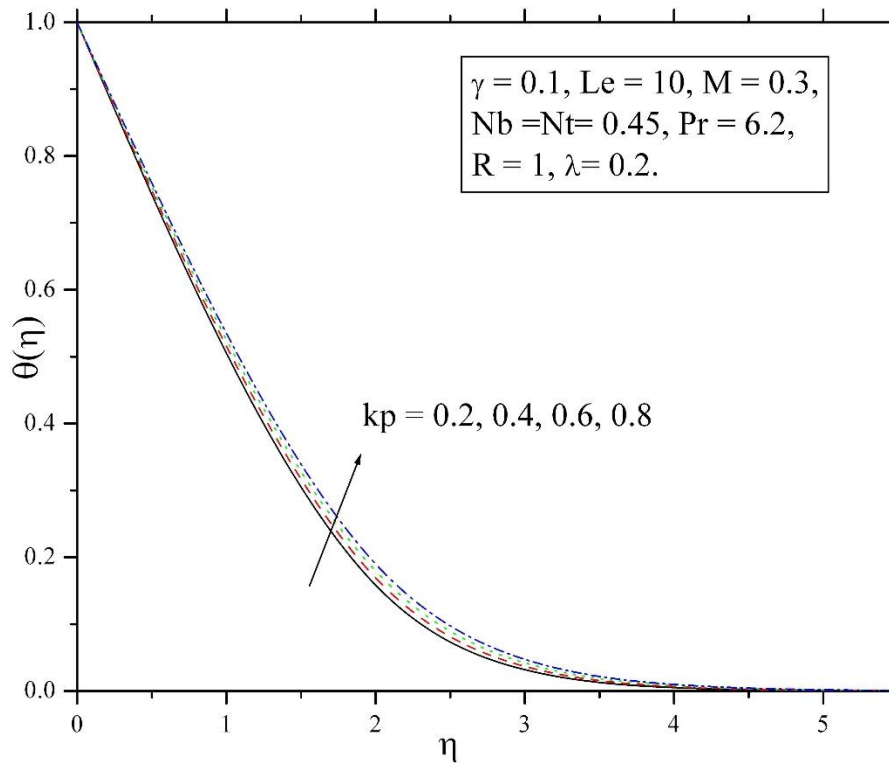


Figure 4: Effect of k_p on $\theta(\eta)$.

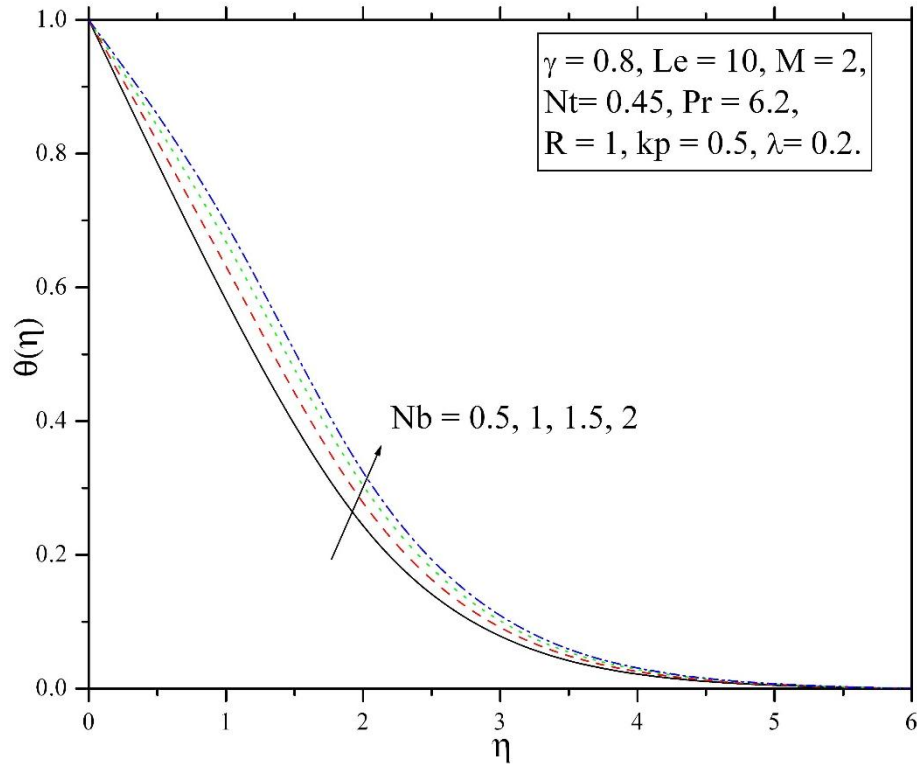


Figure 5: Effect of Nb on $\theta(\eta)$.

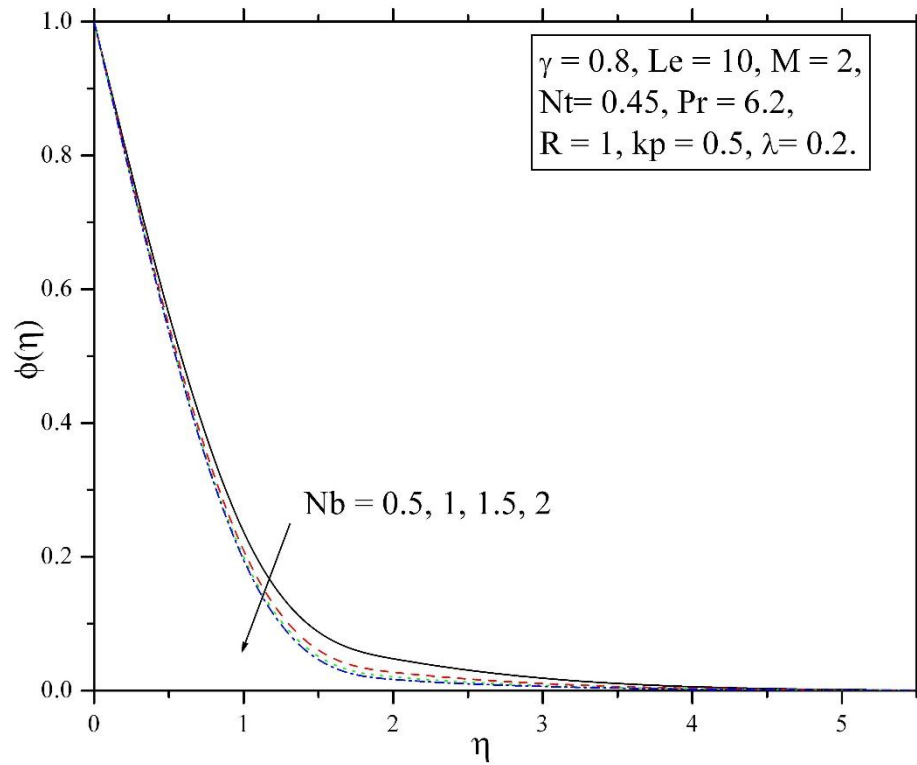


Figure 6: Effect of Nb on $\phi(\eta)$.

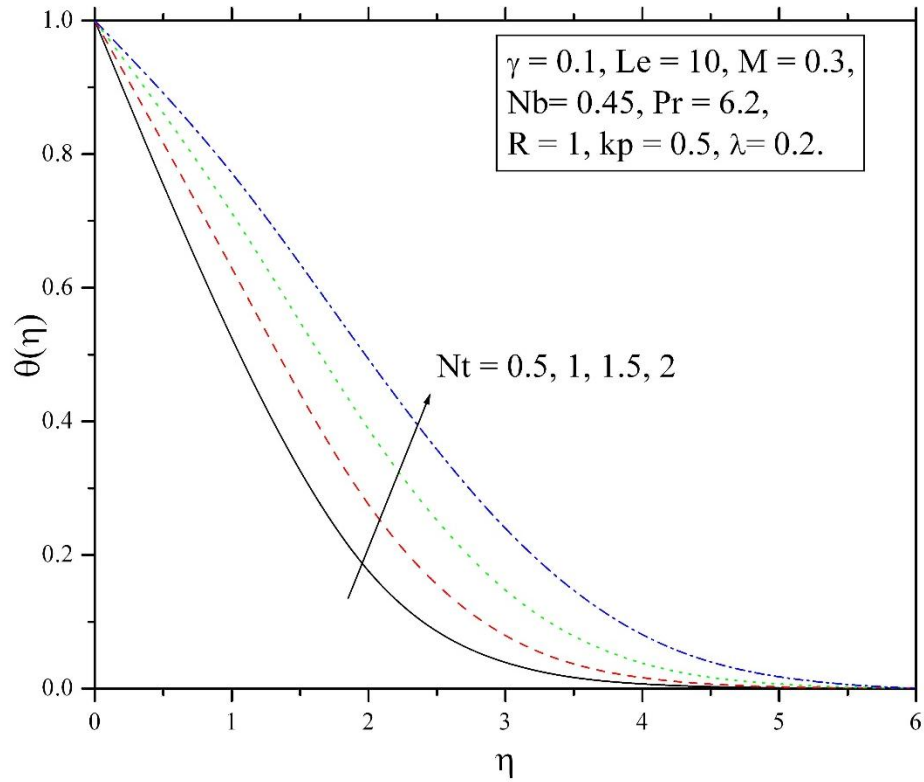


Figure 7: Effect of Nt on $\theta(\eta)$.

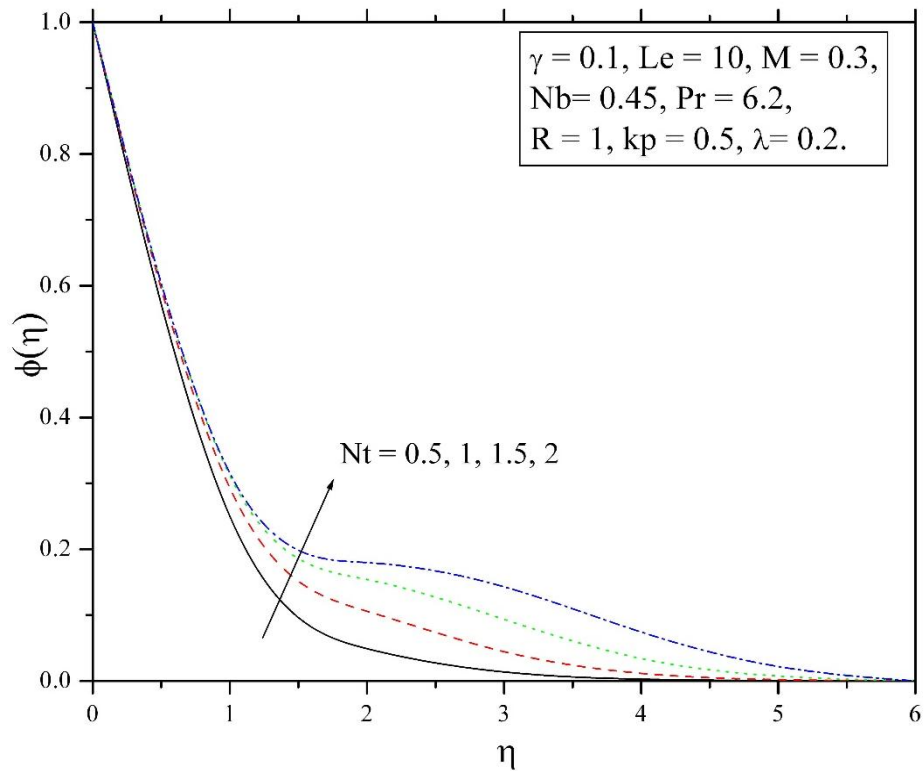


Figure 8: Effect of Nt on $\phi(\eta)$.

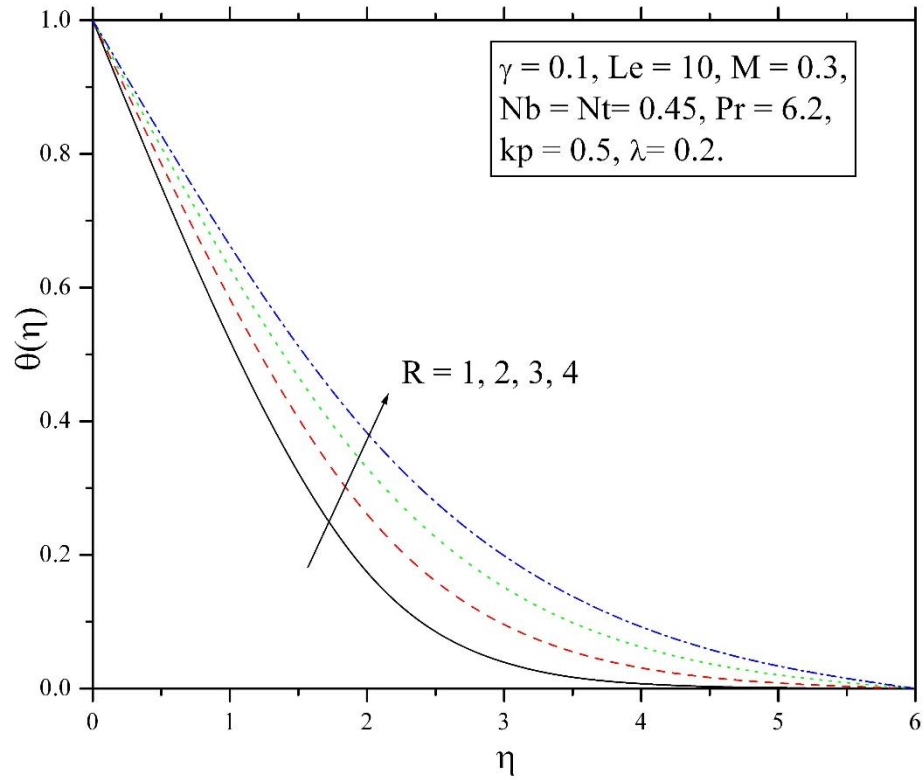


Figure 9: Effect of R on $\theta(\eta)$.

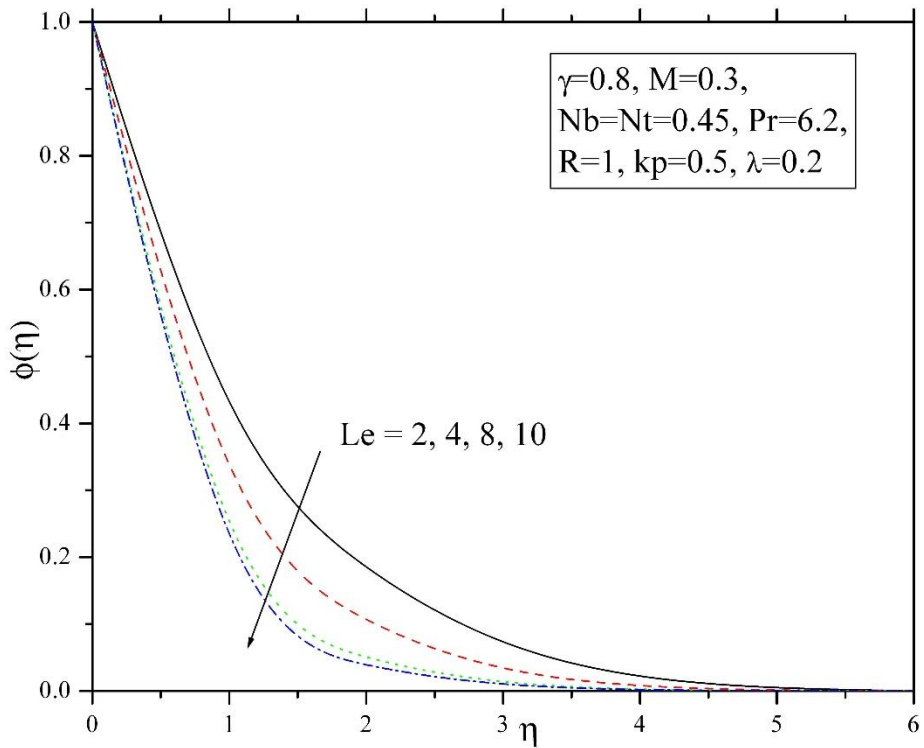


Figure 10: Effect of Nb on $\phi(\eta)$.

III. CONCLUDING REMARKS

The present study numerically investigates the MHD flow, heat transfer, and nanoparticle transport characteristics in a porous medium under the influence of magnetic field, permeability, Brownian motion, thermophoresis, thermal radiation, and Lewis number. The results reveal that increasing the magnetic parameter M significantly suppresses the velocity due to the enhanced Lorentz force, while the temperature increases because of the reduced convective heat transport. Similarly, increasing the permeability parameter increases the resistance offered by the porous medium, leading to a reduction in velocity and an enhancement in temperature. The Brownian motion parameter Nb enhances the temperature distribution and thermal boundary-layer thickness, whereas it reduces the nanoparticle volume-fraction profile due to intensified nanoparticle diffusion. An increase in the thermophoresis parameter Nt enhances both temperature and nanoparticle volume-fraction distributions by promoting nanoparticle migration along the temperature gradient. Furthermore, increasing the thermal radiation parameter R increases the temperature and thermal boundary-layer thickness owing to the additional radiative heat transfer. In contrast, increasing the Lewis number Le decreases the nanoparticle volume-fraction profile because of the reduced nanoparticle mass diffusivity relative to thermal diffusivity. Overall, the results demonstrate that magnetic and porous-medium effects predominantly control momentum transport, while Brownian motion, thermophoresis, thermal radiation, and Lewis number play significant roles in regulating heat and nanoparticle transport. These findings may be useful in the design and optimization of thermal management systems and porous-media applications involving nanofluids.

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