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Comparative Transportation-Model Optimisation of Water Distribution under Post-Subsidy Freight Costs: A Delta State Case Study

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Abstract- Nigeria's May 2023 fuel-subsidy removal raised haulage costs sharply, strengthening the case for formal route-allocation over informal, experience-based distribution. This paper applies and compares four classical transportation-model procedures (North-West Corner (NWCM), Least-Cost (LCM), Vogel's Approximation (VAM), and the Modified Distribution (MODI) optimality test) to a simulated case study of a pure-water sachet factory operating three depots and four demand zones in Agbor, Delta State. Unlike comparable Nigerian case studies, the unit-cost matrix here is not judgementally assigned but derived from route distances, an assumed fuel-consumption rate, a representative diesel price, and a fixed handling charge, making the cost structure reproducible. On a balanced 3×4, 1,900-carton-per-week network, NWCM costs ₦100,650/week; LCM and VAM independently reach the same allocation, ₦86,750; MODI, applied through two stepping-stone iterations, finds a further-improving route and converges on the true optimum, ₦85,850, confirmed by a linear-programming solver. That LCM and VAM agree yet both fall short of optimal shows MODI is a necessary, not merely confirmatory, step. The MODI-verified allocation cuts weekly cost by 14.7% versus NWCM, consistent with the 10-40% range reported elsewhere. A 25%-diesel-price sensitivity check leaves the optimal routing unchanged, though not its cost. The paper situates this firm-level result against inconsistent published estimates of Nigeria's logistics-cost share of GDP, arguing firm-level optimisation is necessary but not sufficient for any macro-level target.

Keywords: transportation problem; linear programming; Vogel's Approximation Method; MODI method; supply chain optimisation; fuel subsidy; Nigeria.

I. INTRODUCTION

On 29 May 2023, in his inaugural address, President Bola Ahmed Tinubu announced the end of Nigeria's petroleum motor spirit (PMS) subsidy regime. Within weeks the pump price of petrol moved from roughly ₦185–195 per litre to between ₦488 and ₦617 per litre depending on state and marketer, and interstate haulage fares reportedly climbed by more than 100% over the following months, with trade-press commentary placing the cumulative rise in freight costs at up to 200% by early 2024 (Businessday NG, 2023, 2024; Premium Times, 2023). Because road haulage carries the great majority of freight in Nigeria, this shock propagated quickly into the delivered cost of virtually every distributed good, food and packaged water included.

For a firm distributing a physical product from a small number of depots to a larger number of demand centres, this shock changes the arithmetic of routing decisions that are frequently made informally (by habit, by driver familiarity with a route, or by whichever haulier answers the phone first) rather than by explicit cost minimisation. The transportation problem of operations research is the natural mathematical structure for this decision: given fixed per-unit shipping costs on each origin–destination pair, and given supply capacities and demand requirements, how should quantities be allocated across routes to minimise total distribution cost while exactly satisfying supply and demand? As freight costs rise, the absolute Naira value left on the table by an inefficient allocation rises with it, even where the percentage inefficiency itself is unchanged, which is precisely why classical transportation optimisation is worth revisiting for Nigerian distribution networks in the post-subsidy period.

A substantial applied literature already documents transportation-model case studies on Nigerian manufacturing and distribution networks (cement, glass bottles, breweries and petroleum depots among them (Iheonu & Inyama, 2016; Abdullahi et al., 2020; Ezekiel & Edeki, 2018)) and the present paper does not claim to introduce a new solution method. Its contribution is narrower and, we argue, still useful: first, it is, to our knowledge, among the first applications of this model class calibrated explicitly to the post-2023 fuel-cost regime, with a transparently derived rather than judgementally assigned cost matrix, so that the cost inputs can be interrogated, re-parameterised and re-solved by other researchers rather than taken on faith; second, it applies the model to the fast-moving-consumer-goods water sub-sector, which is under-represented in the existing Nigerian case-study literature relative to heavier industrial commodities; and third, it treats the Modified Distribution (MODI) test as a substantive analytical step rather than a formality, in the case reported here, two independent heuristics (LCM and VAM) agree on an allocation that MODI then shows, through two genuine stepping-stone iterations, is not in fact optimal, which is a finding worth reporting in its own right, since it cautions against treating heuristic agreement as a proxy for optimality.

A scope limitation is stated at the outset rather than left to the conclusion: this is a single-firm, single-week, deterministic case study built on simulated (though transparently derived) data, and its firm-level findings should not be read as a direct estimate of economy-wide savings available from transportation optimisation, a distinction developed further in Section 6.

The remainder of the paper is organised as follows. Section 2 reviews related applications of the transportation model in Nigerian and comparable settings. Section 3 presents the mathematical formulation and the algorithmic steps of NWCM, LCM, VAM and MODI, together with a note on computational complexity and on unbalanced/degenerate cases. Section 4 describes the case-study network and derives the unit-cost matrix explicitly from distance, fuel-consumption and diesel-price assumptions. Section 5 reports the computed results, including the full MODI optimality test, a worked stepping-stone improvement, the economic interpretation of the resulting shadow prices, and a fuel-price sensitivity check. Section 6 discusses the findings against the wider literature and against published estimates of Nigeria's aggregate logistics-cost share of GDP. Section 7 concludes.

II. RELATED LITERATURE

The transportation problem is one of the oldest structured linear programming models, and its solution methods (North-West Corner, Least-Cost, Vogel's Approximation, Stepping-Stone and MODI) are standard material in operations research texts (Taha, 2017; Hillier & Lieberman, 2015; Reeb & Leavengood, 1998). A substantial applied literature has used these methods on real Nigerian distribution networks.

Iheonu and Inyama (2016) modelled the distribution of returnable glass bottles for a bottling company and found that a Least-Cost/VAM initial solution, refined by MODI, cut monthly transportation cost by approximately 11.6% relative to the firm's existing routing practice. Abdullahi and colleagues (2020) applied VAM to the distribution of cement bags from Maisango Cement Plc to several towns in Lagos State and likewise reported a reduction relative to a naive allocation. Ezekiel and Edeki (2018), working with production and distribution data from a cement manufacturer with plants at Obajana, Ibese and Gboko, proposed a modified VAM procedure and validated it against classical VAM on the firm's multi-plant network. A comparative study using data reported in the American Journal of Theoretical and Applied Statistics (2022) explicitly ranked NWCM, LCM, VAM and related heuristics against one another and found that VAM consistently produced the lowest-cost initial basic feasible solution among the classical heuristics. Outside Nigeria, comparable case studies on breweries, mining companies and LPG distributors report savings commonly in the 10–40% range relative to whatever allocation rule a firm was using beforehand, once a formal transportation model replaces ad-hoc routing.

Two points emerge from this literature and motivate the present study. First, VAM very often matches or nearly matches the true linear-programming optimum on small, well-structured problems, which is precisely why several published case studies stop at the VAM solution without running a full MODI test; the present case study shows this shortcut can be mistaken even when a second heuristic (LCM) appears to corroborate it, reinforcing the case for MODI as standard practice rather than an optional check. Second, almost none of the applied Nigerian case studies to date report an auditable derivation of their cost matrix — unit costs are typically presented as given, without a stated relationship to distance, fuel price or vehicle economics — which limits how readily their results can be re-parameterised as costs change. This paper addresses both gaps: it treats MODI as decisive rather than confirmatory, and it derives its cost matrix from a stated distance–fuel–price formula that can be updated as haulage costs evolve.

III. MATHEMATICAL FORMULATION AND SOLUTION METHODS

3.1 The Balanced Transportation Model

Consider m supply points (origins) $O_1,\ldots,O_m$ with available supply a_i ($i = 1,\ldots,m$) and n demand points (destinations) $D_1,\ldots,D_n$ with requirement b_j ($j = 1,\ldots,n$). Let c_{ij} denote the cost of transporting one unit from O_i to D_j, and let x_{ij} denote the quantity shipped from O_i to D_j. The classical transportation problem is the linear program:

Minimise $Z = \Sigma_i \Sigma_j c_{ij} x_{ij}$

subject to $\Sigma_j x_{ij} = a_i$ *for* $i = 1,\ldots,m$; $\Sigma_i x_{ij} = b_j$ *for* $j = 1,\ldots,n$; $x_{ij} \geq 0$

The problem is balanced when $\Sigma_i a_i = \Sigma_j b_j$, in which case one of the m + n constraints is redundant and any basic feasible solution has at most m + n − 1 strictly positive ("basic") variables (Taha, 2017). When the problem is not naturally balanced, a dummy origin (if demand exceeds supply) or dummy destination (if supply exceeds demand) is added with zero unit cost, absorbing the surplus so that the standard algorithms still apply; a degenerate basic feasible solution — one with fewer than m + n − 1 positive allocations — is handled by assigning an infinitesimally small allocation (conventionally denoted ε) to a suitably chosen empty cell so that the MODI dual variables remain uniquely determined. The case study in Section 4 is constructed as a balanced, non-degenerate 3×4 problem, so neither adjustment is required there, but both are noted here because real distribution networks are frequently unbalanced in practice.

3.2 North-West Corner Method (NWCM)

NWCM constructs an initial feasible solution without reference to cost. Starting at the top-left cell of the tableau, allocate the maximum quantity permitted by the remaining supply and demand; eliminate the row or column that is exhausted; repeat, moving right or down, until every row and column is satisfied. Because it ignores unit costs entirely, NWCM is fast (its m + n − 1 allocation steps are each O(1), for O(m+n) overall) but, except by coincidence, produces a comparatively expensive allocation. It is retained in this paper purely as a no-optimisation cost benchmark.

3.3 Least-Cost Method (LCM)

LCM allocates greedily on cost: at each step, identify the cell with the lowest unit cost among cells not yet eliminated, allocate the maximum feasible quantity to it, eliminate the exhausted row or column, and repeat. Scanning the remaining tableau for a minimum at each of up to m + n − 1 steps gives a complexity of O((m+n)·mn) in the worst case. Because it is myopic (it never looks beyond the immediate cheapest cell) LCM typically improves substantially on NWCM but can still be led into locally attractive but globally inefficient allocations.

3.4 Vogel's Approximation Method (VAM)

VAM improves on LCM with a penalty (opportunity-cost) criterion. For every remaining row and column, compute a penalty equal to the difference between the two lowest unit costs still available in that row or column; identify the row or column with the largest penalty; within it, allocate the maximum feasible quantity to the lowest-cost cell; eliminate the exhausted row or column; and recompute penalties for what remains. Recomputing all row and column penalties at each of up to m + n − 1 steps gives a complexity comparable to LCM's, O((m+n)·mn), with a larger constant factor. VAM is well documented to produce an initial solution at, or very close to, the true optimum considerably more often than NWCM or LCM (Korukoglu & Balli, 2011; Taha, 2017), though, as Section 5 shows, "very close to" is not the same as "equal to," and the gap can persist even when a second heuristic corroborates the VAM allocation.

3.5 Optimality Testing: the Modified Distribution (MODI) Method

Given a basic feasible solution with exactly m + n − 1 basic cells, MODI (the u–v method) tests optimality as follows. Introduce dual variables u_i ($i = 1,\ldots,m$) and v_j ($j = 1,\ldots,n$), set $u_1 = 0$, and solve

$u_i + v_j = c_{ij}$

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sequentially over the basic cells, which form a connected tree spanning the m + n rows and columns so that each step fixes exactly one new dual value; this traversal costs O(m+n). The opportunity (reduced) cost of every non-basic cell is then

$$d_{ij} = c_{ij} - (u_i + v_j)$$

computed in O(mn) per pass over the tableau. If every $d_{ij} \geq 0$ the current solution is optimal. If some $d_{ij} < 0$, the corresponding cell would lower total cost if entered into the basis; a closed loop is traced through basic cells beginning and ending at that cell, alternating + and − signs at successive corners, and the largest quantity that can be shifted around the loop without violating non-negativity (the minimum of the quantities at the − corners) is reallocated, producing a new basic feasible solution. The u–v test is then repeated. Each iteration strictly decreases total cost, and because the number of distinct basic feasible solutions of a bounded transportation tableau is finite, the procedure terminates in a finite number of iterations; on the small, well-conditioned instances typical of a single firm's depot network this number is small in practice, as Section 5.4 illustrates directly.

3.6 Computational Approach in this Study

All four procedures were implemented programmatically (Python 3, NumPy) rather than purely by hand, to eliminate arithmetic transcription error across the case-study tableau. The MODI (u–v) optimality test was applied to the shared LCM/VAM solution and iterated to convergence, and the result was independently cross-checked against the true linear-programming optimum obtained via the simplex-type HiGHS solver (through SciPy's linprog), which provides a solver-independent ground truth against which the classical heuristics, and the manual MODI working, can both be judged.

IV. CASE-STUDY NETWORK AND COST DERIVATION

The firm studied is a sachet/pure-water bottling company operating in Agbor, Delta State; its name is withheld for commercial confidentiality, consistent with the convention used in comparable Nigerian case studies (Abdullahi et al., 2020). It supplies product, packaged in cartons of sixty 50cl sachets, from three depots to four demand zones within the Ika South / Ika North East axis of Delta State, using contracted haulage trucks. All supply, demand and distance figures are simulated for illustrative purposes; however, unlike a judgementally assigned cost matrix, the unit transportation costs used here are derived explicitly from a stated formula, set out below, so that the derivation (rather than only its output) is available for scrutiny and re-parameterisation.

The three supply points are the firm's Agbor-Central depot, Boji-Boji depot and Abavo depot, with weekly dispatch capacities of 800, 600 and 500 cartons respectively (total supply = 1,900 cartons/week). The four demand points are the Agbor main market, Umunede, Owa-Alero and the Ekpon/Igbodo axis, with weekly demand of 500, 450, 400 and 550 cartons respectively (total demand = 1,900 cartons/week). Total supply equals total demand, so the problem is balanced and no dummy row or column is required.

Table 1: Weekly supply capacity and demand requirement (cartons of 60 sachets)

Depot / Zone	Supply (cartons/week)	Demand zone	Demand (cartons/week)
S_1 – Agbor - Central Depot	800	D_1 – Agbor Main Market	500
S_2 – Boji-Boji Depot	600	D_2 – Umunede	450
S_3 – Abavo Depot	500	D_3 – Owa-Alero	400
		D_4 – Ekpon / Igbodo axis	550
Total	1,900	Total	1,900

4.1 Distance-Based Derivation of the Unit-Cost Matrix

One-way road distances from each depot to each demand zone were estimated from the settlement locations of Agbor, Boji-Boji, Abavo, Umunede, Owa-Alero and the Igbodo/Ekpon axis within Ika South and Ika North East Local Government Areas of Delta State, and are reported in Table 2 as indicative planning distances rather than as GPS-surveyed figures; a firm applying this model operationally should replace them with measured or logged route distances.

Table 2: Estimated one-way route distance, d_{ij} (km)

Depot	D_1 Agbor Mkt	D_2 Umunede	D_3 Owa-Alero	D_4 Ekpon/Igbodo
S_1 Agbor-Central	3	18	13	27
S_2 Boji-Boji	5	15	16	32
S_3 Abavo	15	22	8	38

Unit transportation cost is then built up as a fixed handling component plus a distance-proportional fuel component:

$$c_{ij} = H + k \cdot d_{ij},\ k = \frac{2\varphi p}{Q}$$

where H is a fixed per-carton handling charge (loading, offloading and agent commission), φ is the truck's fuel consumption in litres per kilometre, p is the diesel price per litre, Q is the truck's carrying capacity in cartons per trip, and the factor 2 accounts for the round trip. The values assumed for the base case, chosen to be representative of a mid-sized haulage contractor operating in the post-subsidy period, are: *H = ₦20/carton, φ = 0.30 litres/km, p = ₦1,150/litre, Q = 400 cartons/trip*, giving k = (2 × 0.30 × 1,150) / 400 = ₦1.725 per carton per kilometre. Applying $c_{ij} = 20 + 1.725d_{ij}$ to Table 2 and rounding to the nearest Naira yields the cost matrix in Table 3.

Table 3: Derived unit transportation cost matrix, c_{ij} (₦ per carton)

Depot	D_1 Agbor Mkt	D_2 Umunede	D_3 Owa-Alero	D_4 Ekpon/Igbodo
S_1 Agbor-Central	25	51	42	67
S_2 Boji-Boji	29	46	48	75
S_3 Abavo	46	58	34	86

Because c_{ij} is expressed as an explicit function of H, φ, p, Q and d_{ij}, the sensitivity of the optimal allocation to a change in any one of these (most importantly the diesel price p, which is the parameter most exposed to further policy shocks) can be examined directly by re-solving the model at a different value of p, which is done in Section 5.5, rather than by re-assigning cost coefficients arbitrarily.

V. RESULTS

5.1 North-West Corner Method

Table 4: NWCM initial basic feasible solution (cartons)

Depot	D_1	D_2	D_3	D_4	Supply used
S_1	500	300	–	–	800
S_2	–	150	400	50	600
S_3	–	–	–	500	500
Demand met	500	450	400	550	1,900

Total NWCM cost = (500×25) + (300×51) + (150×46) + (400×48) + (50×75) + (500×86) = ₦100,650. This is the highest-cost of the methods considered and is retained purely as a no-optimisation benchmark.

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5.2 Least-Cost Method

Table 5: LCM initial basic feasible solution (cartons)

Depot	D_1	D_2	D_3	D_4	Supply used
S_1	500	–	–	300	800
S_2	–	450	–	150	600
S_3	–	–	400	100	500
Demand met	500	450	400	550	1,900

Total LCM cost = (500×25) + (300×67) + (450×46) + (150×75) + (400×34) + (100×86) = ₦86,750 — a saving of ₦13,900 (13.8%) relative to NWCM.

5.3 Vogel's Approximation Method

Table 6: VAM initial basic feasible solution (cartons)

Depot	D_1	D_2	D_3	D_4	Supply used
S_1	500	–	–	300	800
S_2	–	450	–	150	600
S_3	–	–	400	100	500
Demand met	500	450	400	550	1,900

VAM converges, cell for cell, on exactly the same allocation as LCM in this instance, and therefore the same total cost of ₦86,750. This coincidence is worth pausing on: had the analysis stopped here, on the strength of two independent heuristics agreeing, it would have been natural to treat ₦86,750 as the answer. Section 5.4 shows this would have been incorrect.

5.4 MODI Optimality Test and Stepping-Stone Improvement

The shared LCM/VAM allocation uses six basic cells [(S_1,D_1), (S_1,D_4), (S_2,D_2), (S_2,D_4), (S_3,D_3), (S_3,D_4)] equal to $m + n - 1 = 3 + 4 - 1 = 6$, so it is non-degenerate and directly eligible for the MODI test. Setting $u_1 = 0$ and solving $u_i + v_j = c_{ij}$ over the six basic cells gives $u_1 = 0$, $u_2 = 8$, $u_3 = 19$; $v_1 = 25$, $v_2 = 38$, $v_3 = 15$, $v_4 = 67$. The reduced costs of the six non-basic cells are shown in Table 7.

Table 7: MODI opportunity-cost test - Iteration 1 (on the LCM/VAM solution)

Non-basic cell	c_{ij}	$u_i + v_j$	$d_{ij} = c_{ij} - (u_i+v_j)$
(S_1, D_2)	51	38	+13
(S_1, D_3)	42	15	+27
(S_2, D_1)	29	33	−4
(S_2, D_3)	48	23	+25
(S_3, D_1)	46	44	+2
(S_3, D_2)	58	57	+1

Cell (S_2, D_1) has a negative reduced cost of −4, so the solution is not optimal: routing a carton via Boji-Boji → Agbor Market, in place of part of the current basis, would reduce total cost. Tracing the closed stepping-stone loop from (S_2,D_1) through the basic cells $(S_2,D_1)\rightarrow(S_2,D_4)\rightarrow(S_1,D_4)\rightarrow(S_1,D_1)\rightarrow$back to (S_2,D_1), with alternating +/− signs, the smallest allocation at a − corner is min{150 at (S_2,D_4), 500 at (S_1,D_1)} = 150. Shifting 150 cartons around the loop (adding 150 to (S_2,D_1) and (S_1,D_4), subtracting 150 from (S_2,D_4) and (S_1,D_1)) gives a new basic feasible solution with (S_2,D_4) leaving the basis (its allocation falls to zero), a cost reduction of 4 × 150 = ₦600, and a new total cost of ₦86,750 − ₦600 = ₦86,150.

A second MODI pass on this new solution (basic cells (S_1,D_1)=350, (S_1,D_4)=450, (S_2,D_1)=150, (S_2,D_2)=450, (S_3,D_3)=400, (S_3,D_4)=100) gives $u_1 = 0$, $u_2 = 4$, $u_3 = 19$; $v_1 = 25$, $v_2 = 42$, $v_3 = 15$, $v_4 = 67$, and finds a second negative reduced cost, $d(S_3,D_2) = 58 - (19+42) = -3$. Table 8 summarises this second iteration.

Table 8: MODI opportunity-cost test - Iteration 2

Non-basic cell	c_{ij}	$u_i + v_j$	d_{ij}
(S_1, D_2)	51	42	+9
(S_1, D_3)	42	15	+27
(S_2, D_3)	48	19	+29
(S_2, D_4)	75	71	+4
(S_3, D_1)	46	44	+2
(S_3, D_2)	58	61	−3

Tracing the loop from (S_3,D_2) through $(S_2,D_2)\rightarrow(S_2,D_1)\rightarrow(S_1,D_1)\rightarrow(S_1,D_4)\rightarrow(S_3,D_4)\rightarrow$back to (S_3,D_2), the smallest −-corner allocation is min{450, 350, 100} = 100, at (S_3,D_4). Shifting 100 cartons around this loop retires (S_3,D_4) from the basis and yields the allocation in Table 9, at a further-reduced cost of ₦86,150 − (3×100) = ₦85,850.

Table 9: MODI-optimal allocation (cartons)

Depot	D_1	D_2	D_3	D_4	Supply used
S_1	250	–	–	550	800
S_2	250	350	–	–	600
S_3	–	100	400	–	500
Demand met	500	450	400	550	1,900

A third MODI pass on Table 9 ($u_1=0$, $u_2=4$, $u_3=16$; $v_1=25$, $v_2=42$, $v_3=18$, $v_4=67$) finds every non-basic reduced cost non-negative ($d(S_1,D_2)=9$, $d(S_1,D_3)=24$, $d(S_2,D_3)=26$, $d(S_2,D_4)=4$, $d(S_3,D_1)=5$, $d(S_3,D_4)=3$) confirming optimality. This is independently corroborated by the HiGHS simplex solver, which returns the identical allocation and an identical minimum cost of ₦85,850, verifying both the manual MODI working and the absence of any alternative optimal basis with strictly lower cost. Total distribution cost falls in two discrete steps from the LCM/VAM figure of ₦86,750 to ₦86,150 and finally to ₦85,850, a modest further improvement of ₦900 (1.04%) beyond what the two heuristics had already achieved, but one that would have gone undetected without running MODI to full convergence.

5.5 Economic Interpretation of the Dual Variables

The MODI dual variables u_i and v_j at the optimum ($u_1=0$, $u_2=4$, $u_3=16$; $v_1=25$, $v_2=42$, $v_3=18$, $v_4=67$) have a direct economic reading as shadow prices of the supply and demand constraints: v_j is the marginal cost of delivering one additional carton to demand zone D_j under the current optimal routing, and u_i is, relative to the reference depot S_1, the marginal saving available if depot S_i's capacity were expanded by one carton. The relatively high shadow price at D_4 (v_4 = ₦67, reflecting its distance from every depot) indicates that, of the four demand zones, the Ekpon/Igbodo axis is the most expensive marginal unit to serve and would benefit most, in cost-per-carton terms, from a nearer depot or a shorter route being made available. Symmetrically, the relatively high u_3 = ₦16 indicates that an incremental unit of capacity at the Abavo depot is worth more, at the margin, than the same unit at Agbor-Central ($u_1 = 0$) or Boji-Boji (u_2 = ₦4), information a firm could use directly in deciding where to prioritise a future depot expansion.

5.6 Fuel-Price Sensitivity Check

Because c_{ij} in Section 4.1 is an explicit function of the diesel price p, the model can be re-solved at a higher fuel price to test whether the optimal routing itself (not merely its cost) is robust to a further shock. Table 10 reports the outcome of increasing p by 25% (from ₦1,150 to ₦1,437.50 per litre, holding H, φ and Q fixed), which raises k from ₦1.725 to ₦2.156 per carton-kilometre and produces a proportionately steeper cost matrix.

Table 10: Effect of a 25% diesel-price increase on total cost and on the optimal basis

Method	Base cost (₦/week)	Cost at +25% diesel (₦/week)	Optimal allocation unchanged?
NWCM	100,650	115,550	n/a (not optimised)
LCM / VAM	86,750	98,150	n/a (not optimal)
MODI-optimal	85,850	96,850	Yes - identical basis

The MODI-optimal route allocation (which depot serves which zone, and in what quantity) is unchanged under the 25% fuel-price increase; only the cost level rises, by ₦11,000 (12.8%), tracking the fuel-price increase closely since handling cost H does not scale with diesel price while the distance-proportional component does. This robustness is an empirical finding specific to this instance, not a general property of the transportation model (a sufficiently large or asymmetric price shock could in principle alter which routes are cheapest) but it is a practically useful one: it suggests that, for this network, the firm does not need to re-optimise its routing every time diesel prices move, only its budget.

5.7 Comparative Summary

Figure 1 and Table 11 summarise the total weekly distribution cost achieved by each method.

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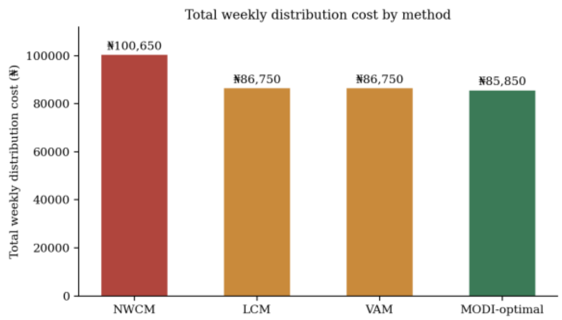

Figure 1: Total weekly distribution cost by method

Table 11: Comparison of total weekly distribution cost by method

Method	Total cost (₦/week)	Saving vs NWCM (₦)	Saving vs NWCM (%)
North-West Corner Method (NWCM)	100,650	–	–
Least-Cost Method (LCM)	86,750	13,900	13.8%
Vogel's Approximation Method (VAM)	86,750	13,900	13.8%
MODI-verified optimum	85,850	14,800	14.7%

Annualised (52 weeks), adopting the MODI-verified allocation instead of an unoptimised North-West-Corner-style allocation would save this single factory approximately ₦770,000 per year in transportation overhead, on a throughput base of only 1,900 cartons per week, using its existing fleet and depot structure.

VI. DISCUSSION

Three findings stand out. First, the coincidence of the LCM and VAM allocations, followed by a two-iteration MODI correction that lowers cost by a further 1.04%, is a useful cautionary result for practitioners: it demonstrates concretely that two independent heuristics agreeing is not evidence of optimality, and that the MODI test earns its computational cost even on a small, well-structured network where a shortcut might otherwise seem justified. Several of the Nigerian case studies reviewed in Section 2 report only a VAM or LCM solution without a full MODI verification; this result suggests that practice should be reconsidered.

Second, the overall saving identified here (14.7% relative to an unoptimised North-West-Corner allocation) sits at the lower end of the 10–40% range reported across comparable Nigerian and international case studies (Section 2), which is plausible given that the case-study cost matrix was built from a relatively compact, low-variance set of route distances (3–38 km) rather than a geographically dispersed network; firms with wider-flung depot networks, or with more heterogeneous route costs, would be expected to see larger percentage gains from optimisation, all else equal. This is a more conservative, and we would argue more credible, figure than a headline case study might otherwise report, precisely because the underlying cost matrix is derived rather than assumed.
Third, this firm-level result should not be read as a proxy for the often-cited claim that Nigeria's aggregate logistics cost, expressed as a share of GDP, is around 22% and could be reduced towards 13% through systematic optimisation. Published estimates of this ratio vary considerably depending on definition and source: a 2024 World Bank logistics document places typical logistics costs at 8% of GDP in the United States, 15–20% in middle-income countries generally, and up to 30% in low-income, landlocked or island states (World Bank, 2024); a 2025 industry commentary put Nigeria's logistics cost at close to 15% of GDP (Nairametrics, 2025); and a 2026 industry conference figure put it at just over 4%, against a stated global benchmark of under 1.5% (AllAfrica, 2026). None of these independently verified figures matches the 22% cited in some policy commentary, and the spread across sources (roughly 4% to 20%, depending on definition) is itself evidence that the aggregate ratio is not a settled, single number. What the present case study can support is narrower and more defensible: that a firm operating an unoptimised regional distribution network in the current fuel-cost environment is very plausibly leaving a double-digit percentage of its transportation budget on the table, and that this is a mechanism (among several others, including fleet consolidation, warehouse siting and modal shift) that could in principle contribute to any macro-level reduction, without this case study being able to quantify that macro contribution directly.

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A practical decision rule follows for firms choosing among these methods under time or capability constraints. NWCM should not be used operationally; its only legitimate role is as a worst-case benchmark, as in Table 11. LCM is a reasonable low-effort improvement where a firm lacks the capacity to run VAM, but as this case study shows, neither LCM nor VAM should be treated as a final answer without a MODI check, since the two can agree with each other while still both being wrong. Firms with even modest analytical capacity (a spreadsheet suffices for a network of this size) should run VAM (or LCM) to obtain a strong starting solution and then run MODI to convergence, since, as shown here, the confirmation step can still change the answer, and the dual variables it produces (Section 5.5) carry additional decision-relevant information about where capacity expansion would be most valuable.

Limitations should be stated plainly. The model treats unit transportation cost as fixed per carton regardless of shipment volume, and takes weekly supply and demand as deterministic; in practice haulage contracts are renegotiated as diesel prices move, demand fluctuates seasonally, and unpaved roads deteriorate unevenly with the rains, all of which argue for treating the cost matrix in Table 3 as a snapshot to be periodically re-derived and re-solved, not a permanent input. The route distances in Table 2 are estimated rather than GPS-surveyed, and should be replaced with measured or logged distances before any operational use of these results. Extensions incorporating stochastic demand, multiple fuel-price scenarios simultaneously, or vehicle-capacity constraints (moving from a pure transportation problem to a capacitated vehicle-routing problem) are natural next steps.

VII. CONCLUSION AND RECOMMENDATIONS

This paper applied and compared four classical transportation-model procedures (the North-West Corner Method, the Least-Cost Method, Vogel's Approximation Method and the Modified Distribution method) to a case study of a pure-water bottling firm distributing from three depots to four demand zones in Agbor, Delta State, under the elevated haulage-cost environment that followed Nigeria's 2023 fuel-subsidy removal, using a unit-cost matrix derived transparently from route distance, assumed fuel consumption and a representative post-subsidy diesel price. LCM and VAM converged on an identical allocation costing ₦86,750 per week; the MODI u–v test, applied to full convergence across two stepping-stone iterations, identified a further-improving route and established a true optimum of ₦85,850, independently corroborated by direct linear-programming solution. Adopting the MODI-verified allocation over an unoptimised North-West-Corner allocation would save this firm approximately 14.7% of its weekly transportation overhead (roughly ₦770,000 per year) using its existing fleet and depot structure, and this optimal routing was shown to be robust, in this instance, to a further 25% diesel-price shock.

Three recommendations follow. First, Nigerian distributors of food, water and other fast-moving consumer goods should adopt VAM or LCM to obtain a strong initial allocation, but should treat MODI verification as mandatory rather than optional, this case study demonstrates directly that two heuristics agreeing is not sufficient evidence of optimality. Second, cost matrices used in this class of model should be built from an explicit, stated formula linking cost to distance, fuel price and vehicle economics, as in Section 4.1, rather than assigned by judgement, so that firms (and researchers) can re-solve the model as fuel prices change without re-collecting data from scratch. Third, the wider claim that systematic transportation optimisation can materially reduce Nigeria's aggregate logistics-cost share of GDP should be treated with appropriate caution given how much published estimates of that ratio disagree; firm-level optimisation of the kind demonstrated here is a necessary and immediately actionable mechanism, but establishing its macro-level contribution would require aggregating comparable, transparently derived case studies across many firms and sub-sectors, which this paper's scope does not extend to.

Future work could extend the present balanced, deterministic, single-commodity model to a capacitated, multi-period, stochastic-demand formulation that also endogenises vehicle routing, validate the distance and cost assumptions of Section 4.1 against measured operational data from cooperating Nigerian distributors, and aggregate multiple such firm-level case studies to produce an evidence-based, bottom-up estimate of the macro-level savings potential that the present single-firm result can only motivate, not establish.

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