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Finite Modular Coordinate Geometry and Algebraic Structures of Hypercubes

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Abstract- ***This paper develops a unified mathematical framework for finite modular coordinate spaces, beginning with the elementary but structurally rich relation between Z_2 and the vertices of a square, and extending it to cubes, n-dimensional hypercubes, ternary coordinate spaces, finite vector spaces, graph structures, coding theory, digital mathematics and quantum-information indexing. The principal coordinate object is C_{q,n}=Z_q^n, with the binary case C_{2,n}=F_2^n providing the vertex set of the n-dimensional hypercube. We establish results for vertex counts, adjacency, Hamming distance, Euclidean distance, Hamming spheres, faces, shortest paths and affine transformations. The q-ary extension gives a common language for binary and ternary coordinate systems. Applications are developed to binary digital images and voxels, error-correcting codes, hypercube computer networks, Boolean operations, and computational-basis indexing of qubits and qutrits. A mathematically precise bridge to quantum information is obtained through Pauli-X and generalized Weyl translation operators. We then propose a research programme for modular cubical algebra, in which graph-compatible operations, affine transformations and finite near-ring structures are studied together. The paper distinguishes established results from proposed research directions and identifies several problems suitable for further investigation.***

Keywords: modular coordinate geometry; finite fields; F_2^n; F_3^n; hypercube; Hamming distance; coding theory; digital algebra; graph networks; Pauli-X; qutrits; modular transformations; near-rings.

I. INTRODUCTION

Finite modular arithmetic provides a natural algebraic language for systems in which coordinates have a finite number of states. The binary system has two symbols, 0 and 1, and therefore appears naturally in digital computation, Boolean algebra, coding theory and the combinatorics of hypercubes. The set Z_2={0,1}, equipped with addition and multiplication modulo 2, is the field F_2. Its Cartesian powers F_2^n contain 2^n vectors and can be identified with the vertex labels of the n-dimensional unit hypercube. Thus a square has four binary coordinate vertices, a cube has eight, and the n-cube has 2^n vertices.

The ternary system Z_3={0,1,2} is also a field, F_3, and its powers F_3^n provide finite ternary coordinate spaces with 3^n points. These spaces are not the vertex sets of ordinary Euclidean cubes when n>1, but they are the natural coordinate spaces for q-ary combinatorics and finite geometry. The distinction is important: the same symbols can label Euclidean points and algebraic vectors, but the operations and metrics need not be the same.

The purpose of this paper is to combine these observations into a broader framework. The binary case is treated as a central model and is then generalized to q-ary coordinates. The framework is connected to graph theory through the hypercube graph Q_n, to coding theory through Hamming geometry, to digital mathematics through XOR and Boolean functions, and to quantum information through computational-basis labels. The quantum section is deliberately formulated as a representation bridge: F_2^n is not identified with the complex Hilbert space of n qubits; rather, it indexes computational basis states and modular translations act through unitary operators.

A further goal is to formulate a research programme rather than to present classical facts as new discoveries. The basic correspondence between F_2^n and hypercube vertices is well known. The potentially novel direction is to investigate algebraic operations that are simultaneously compatible with modular coordinates, graph structure and applications. This motivates the term modular cubical algebra for the proposed family of structures.

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II. PRELIMINARIES

2.1 Modular arithmetic

For a positive integer q, Z_q denotes the residue classes modulo q. Addition and multiplication are defined by reducing integer results modulo q. If q is prime, Z_q is a field, conventionally denoted F_q. If q is composite, Z_q is a ring but generally not a field.

a +_q b = (a+b) mod q, a ·_q b = (ab) mod q.

2.2 Cartesian powers and finite vector spaces

For a finite alphabet A with q symbols, A^n is the set of ordered n-tuples over A. When A=F_q, the coordinatewise operations make F_q^n an n-dimensional vector space over F_q, containing q^n elements. For n>1 it is a vector space, not a field under coordinatewise addition and multiplication, because nonzero vectors can have zero coordinates and therefore need not possess multiplicative inverses.

Theorem 1 (Cardinality). For every finite alphabet of size q, |Z_q^n|=q^n.

Proof. Each of the n coordinates has q independent choices. The product rule for finite Cartesian sets therefore gives q·q·...·q=q^n choices.

Theorem 2 (Vector-space structure). If q is prime, F_q^n is an n-dimensional vector space over F_q and has q^n elements.

Proof. Coordinatewise addition and scalar multiplication satisfy the vector-space axioms because they are inherited from the field F_q. The standard unit vectors form a basis, so the dimension is n. Each vector is uniquely determined by n field coordinates, giving q^n elements.

III. BINARY MODULAR COORDINATE GEOMETRY

3.1 The square

The four vertices of the unit square can be labelled by the elements of F_2^2:

Vector	Euclidean point
(0,0)	(0,0)
(0,1)	(0,1)
(1,0)	(1,0)
(1,1)	(1,1)

The algebraic operations are coordinatewise modulo 2. For example, (1,0)+(1,1)=(0,1). This is the XOR operation on two-bit strings.

Theorem 3 (Square correspondence). The vertex set of the unit square is naturally identified with F_2^2.

Proof. A vertex of the unit square has each coordinate equal to 0 or 1. Hence its coordinate pair is an element of {0,1}^2=F_2^2, and every element of F_2^2 gives one such vertex.

3.2 The cube

Similarly, the unit cube has eight vertices and can be labelled by F_2^3. The eight vectors are 000,001,010,011,100,101,110,111.

Theorem 4 (Cube correspondence). The vertex set of the unit cube is naturally identified with F_2^3.

Proof. Every cube vertex has three binary coordinates, giving an element of F_2^3. Conversely, each binary triple determines a unique cube vertex.

3.3 The n-dimensional hypercube

Theorem 5 (Hypercube vertex theorem). The vertices of the n-dimensional binary hypercube are naturally represented by F_2^n, and therefore the hypercube has 2^n vertices.

Proof. A hypercube vertex is an n-tuple whose coordinates are 0 or 1. Thus its coordinate vector belongs to F_2^n. By Theorem 1, this set contains 2^n vectors.

n	F_2^n cardinality	Geometric object
1	2	line segment endpoints
2	4	square vertices
3	8	cube vertices
4	16	4-cube vertices
5	32	5-cube vertices
6	64	6-cube vertices

IV. TERNARY AND Q-ARY COORDINATE GEOMETRY

For q=3, Z_3=F_3 and Z_3^2 contains nine points arranged as a 3×3 modular coordinate grid. Z_3^3 contains 27 points. These are best viewed as finite

ternary coordinate spaces rather than as the vertices of an ordinary cube.

Space	Number of points	Interpretation
F_2^2	4	binary square labels
F_2^3	8	binary cube labels
F_3^2	9	ternary coordinate grid
F_3^3	27	ternary 3-dimensional grid
F_q^n	q^n	q-ary coordinate space

Theorem 6 (q-ary cardinality). The finite coordinate space F_q^n contains q^n points.
Proof. There are q choices in each of n coordinates. Independent choices give q^n vectors.

For q=2, the coordinate alphabet coincides with the endpoints of a unit interval and the resulting point set is exactly the hypercube vertex set. For q>2, the same algebraic construction produces a q-ary grid with more than two values per coordinate.

4.1 Hamming geometry
For x,y in Z_q^n, the Hamming distance d_H(x,y) is the number of coordinates in which x and y differ. It is a discrete metric and is particularly natural for digital and coding applications.

Theorem 7 (Hamming sphere size). In F_q^n, the number of vectors at Hamming distance k from a fixed vector is C(n,k)(q−1)^k.

Proof. Choose the k coordinates that will differ in C(n,k) ways. In each selected coordinate there are q−1 alternatives, while all remaining coordinates are fixed. Multiplication gives C(n,k)(q−1)^k.

For q=2 this becomes C(n,k), while for q=3 it becomes C(n,k)2^k. This provides a direct quantitative bridge between binary and ternary systems.

V. HYPERCUBE GRAPHS

The n-dimensional hypercube graph Q_n has vertex set F_2^n. Two vertices are adjacent precisely when they differ in exactly one coordinate.

Theorem 8 (Adjacency characterization). For x,y in F_2^n, x and y are adjacent in Q_n if and only if d_H(x,y)=1.

Proof. This is the defining construction of Q_n: one edge changes exactly one binary coordinate. Conversely, if the Hamming distance is one, exactly one coordinate differs, so the corresponding vertices are joined by an edge.

Theorem 9 (Shortest-path distance). For x,y in Q_n, the graph distance d_Q(x,y) equals d_H(x,y).

Proof. Every edge changes one coordinate, so at least d_H(x,y) edges are required to change all differing coordinates. Changing those coordinates one at a time constructs a path of exactly d_H(x,y) edges. Hence the lower bound is attained.

Theorem 10 (Degree and edge count). Every vertex of Q_n has degree n and Q_n has n2^{n−1} edges.

Proof. From any vertex, one may flip any one of the n coordinates, giving n distinct neighbors. The handshaking lemma gives 2|E|=n2^n, hence |E|=n2^{n−1}.

VI. EUCLIDEAN AND HAMMING DISTANCE

When binary vectors are embedded as points in R^n, Euclidean and Hamming distances are related but are not identical metrics. If x,y differ in k binary coordinates, each differing coordinate contributes 1 to the squared Euclidean distance.

Theorem 11 (Metric relation for binary vertices). For x,y in F_2^n viewed as Euclidean points, d_E(x,y)=sqrt(d_H(x,y)).

Proof. If the Hamming distance is k, then exactly k coordinate differences have absolute value 1 and all

other coordinate differences are zero. Thus d_E^2=k, giving d_E=sqrt(k).

This relation is useful because it shows that the combinatorial geometry of the hypercube and its Euclidean embedding encode the same number k in two different ways: graph distance is k while Euclidean distance is sqrt(k).

VII. FACES AND COMBINATORIAL STRUCTURE

Theorem 12 (Number of k-faces). The n-cube has C(n,k)2^{n−k} faces of dimension k.
Proof. Choose the k coordinates that vary: C(n,k) choices. The remaining n−k coordinates are fixed to 0 or 1, giving 2^{n−k} choices. Each choice defines one k-face.

Dimension k	Number of k-faces in Q_n
0	2^n
1	n2^{n−1}
2	C(n,2)2^{n−2}
n−1	2n
n	1

VIII. AFFINE TRANSFORMATIONS OVER FINITE COORDINATE SPACES

A central class of finite transformations is given by affine maps T(x)=Ax+b, where A is an n×n matrix over F_q and b is a translation vector. When A is invertible, T is a bijection.

Theorem 13 (Affine bijection). If A∈GL(n,q), then T(x)=Ax+b is a bijection on F_q^n.
Proof. The inverse is T^{-1}(y)=A^{-1}(y−b), which is well-defined because A is invertible.

Affine transformations provide a common language for finite geometry, digital transformations and algebraic coding operations. Not every affine map preserves Hamming distance or hypercube adjacency; preservation is an additional property that must be checked.

Theorem 14 (Translation). For every a∈F_q^n, the map T_a(x)=x+a is a bijection of F_q^n.

Proof. Its inverse is T_{−a}, because (x+a)−a=x.

IX. BOOLEAN ALGEBRA AND DIGITAL MATHEMATICS

For binary vectors, addition modulo 2 is exactly bitwise XOR. Thus F_2^n provides an algebraic model for n-bit digital words.

x	y	x+y mod 2	XOR
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

Theorem 15 (XOR as vector addition). Bitwise XOR of binary words is identical to coordinatewise addition in F_2^n.

Proof. In each coordinate, the XOR truth table is precisely addition modulo 2: 0⊕0=0, 0⊕1=1, 1⊕0=1, and 1⊕1=0. Applying this coordinatewise proves the result.

This permits digital operations to be studied using linear algebra. A binary image with N binary pixels can be represented by a vector in F_2^N; a binary voxel volume can be represented in the same way after flattening its coordinates.

Theorem 16 (Binary data model). Any binary data object containing N independently encoded binary components can be represented as a point of F_2^N.
Proof. Each component is either 0 or 1, so the ordered collection is an N-tuple in F_2^N. The representation is one-to-one once an ordering of components is fixed.

X. DIGITAL IMAGE AND VOXEL APPLICATIONS

Consider a binary image with four pixels encoded as x=(1,0,1,1). Another image is y=(1,1,0,1). Their difference pattern is x+y=(0,1,1,0). The Hamming distance is 2, so two pixel positions differ.
d_H(x,y)=2.

For a three-dimensional binary voxel object, the same construction represents occupancy states by F_2^N.

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Local changes, global XOR masks and mismatch counts can therefore be expressed algebraically. This does not replace geometric image models; it supplies a finite-state algebraic layer useful for classification, masking and error analysis.

XI. CODING THEORY

A binary block of length n is a vertex of Q_n. An error pattern e∈F_2^n changes a codeword c into r=c+e. Therefore transmission with binary errors is naturally described by translations in F_2^n.

Theorem 17 (Error translation). For binary transmission, if c is the transmitted word and e is the error vector, then r=c+e and d_H(c,r)=wt(e), where wt(e) is the number of ones in e.

Proof. Since r+c=e in characteristic two, c and r differ exactly in the coordinates where e has a one. Hence their Hamming distance equals the Hamming weight of e.

A code C⊂F_2^n is therefore a set of selected hypercube vertices. Minimum Hamming distance determines the classical error-detection and error-correction capability of the code. In the q-ary case the same geometry uses F_q^n and the sphere-size formula of Theorem 7.

XII. HYPERCUBE COMPUTER NETWORKS

Hypercube interconnection networks assign one binary n-tuple to each processor. A communication link is present when processor labels differ in one bit. The graph distance theorem therefore gives an optimal routing principle: the number of required links is exactly the Hamming distance between the endpoint labels.

Theorem 18 (Optimal hypercube routing). In Q_n, a route between x and y can be constructed in d_H(x,y) steps and no shorter route exists.

Proof. This follows directly from Theorem 9. Flip each differing coordinate once; every route must change every differing coordinate at least once.

XIII. QUANTUM-INFORMATION APPLICATION

The connection with quantum information is a representation of modular labels, not an equality between finite vector spaces and Hilbert spaces. For n qubits, the computational basis consists of 2^n states |x⟩ indexed by x∈F_2^n.

$|x\rangle = |x_1\rangle \otimes \cdots \otimes |x_n\rangle, \quad x \in F_2^n.$

The Pauli-X operator satisfies X|0⟩=|1⟩ and X|1⟩=|0⟩. Hence products of X operators implement coordinate flips on binary labels.

Theorem 19 (Pauli-X translation theorem). For a,x∈F_2^n, (X^{a_1}⊗⋯⊗X^{a_n})|x⟩=|x+a⟩.

Proof. For coordinate i, X^{a_i}|x_i⟩ equals |x_i+a_i mod 2⟩: if a_i=0 the coordinate is unchanged, and if a_i=1 it is flipped. Taking the tensor product over all coordinates produces the basis state whose label is x+a.

Thus the additive translation group of F_2^n is represented by multi-qubit bit-flip operations. This provides a precise connection between hypercube translations and quantum gates. The underlying quantum state space remains the complex Hilbert space (C^2)^{⊗n}; F_2^n serves as the finite label set for computational basis states.

13.1 Ternary and qudit extension

For q-level systems, let ω=exp(2πi/q) and define generalized shift and phase operators on computational basis states by X|j⟩=|j+1 mod q⟩ and Z|j⟩=ω^j|j⟩. Then X^a translates the q-ary label by a. For n systems, tensor products of such shifts represent translations of Z_q^n.

Theorem 20 (q-ary shift representation). For a,x∈Z_q^n, (X^{a_1}⊗⋯⊗X^{a_n})|x⟩=|x+a⟩, with addition modulo q.

Proof. Apply the one-system identity X^{a_i}|x_i⟩=|x_i+a_i mod q⟩ in each tensor factor. The resulting tensor product is exactly the computational basis state labelled by x+a.

For q=3 this gives a natural algebraic indexing of qutrit computational basis states by F_3^n. Again, the finite

modular space is a label space embedded conceptually into a complex quantum model, rather than the quantum Hilbert space itself.

XIV. A PROPOSED MODULAR CUBICAL ALGEBRA

The classical correspondence $F_2^n \leftrightarrow V(Q_n)$ suggests a broader research structure. Define a modular cubical system as a triple $M=(F_q^n,+,\Gamma)$, where Γ is a graph or adjacency relation on F_q^n. For $q=2$ and Γ defined by Hamming distance one, Γ is Q_n.

A modular cubical algebra may then be studied by imposing compatibility between algebraic operations and graph transformations. One possible research family is obtained by considering binary operations * on F_q^n for which $(F_q^n,+,*)$ satisfies selected near-ring or ring axioms, while multiplication-induced maps interact with Γ in a controlled way.

Theorem 21 (Finite near-ring extension principle). If an operation * on F_2^n makes $(F_2^n,+,*)$ a near-ring under a chosen near-ring convention, then the resulting near-ring has order 2^n.

Proof. The additive group is F_2^n, which contains 2^n elements. The near-ring axioms concern the operations and do not change the underlying set, so the order is 2^n.

The theorem is elementary but opens a classification problem: which near-ring multiplications on a binary coordinate space are compatible with a chosen graph structure? This is substantially different from claiming that every hypercube automatically carries a nonstandard near-ring structure. Compatibility must be constructed and verified.

14.1 Graph-compatible transformations

A useful research question is to define a multiplication or family of transformations for which left or right translations preserve selected graph features. For example, one may require that maps $x \mapsto a*x+b$ preserve adjacency, preserve Hamming spheres, or induce graph endomorphisms. Such conditions can be used to define subclasses of modular cubical algebras.

Another direction is to compare affine groups, graph automorphism groups and algebraic automorphism groups. Coordinate permutations and coordinate flips generate many symmetries of Q_n, while general linear transformations preserve vector-space structure but need not preserve hypercube adjacency. This separation creates a meaningful classification problem.

XV. UNIFIED Q-ARY FRAMEWORK

Theorem 22 (Unified coordinate framework). Let A be a finite alphabet of q symbols and let $C_{q,n}=A^n$. Then $C_{q,n}$ simultaneously supports coordinate counting, Hamming geometry, and—when $A=F_q$—finite vector-space algebra.

Proof. The q^n coordinate count is independent of algebraic structure. Hamming distance is defined by coordinate disagreement. If $A=F_q$, coordinatewise field operations additionally make the set a vector space.

System	Alphabet size	n-coordinate states	Primary geometry
Binary	2	2^n	hypercube/Hamming
Ternary	3	3^n	ternary grid/Hamming
q-ary	q	q^n	q-ary Hamming space

The framework therefore separates three layers: (i) coordinate combinatorics, (ii) algebraic structure, and (iii) application-specific interpretation. Keeping these layers distinct avoids category errors while allowing them to interact.

XVI. EXAMPLES

Example 1: Square translation
Let x=(1,0), a=(1,1). Then x+a=(0,1). Geometrically, the labels move from one square vertex to the opposite vertex. Algebraically, this is a translation in F_2^2.
Example 2: Cube path
Let x=000 and y=111. Then $d_H(x,y)=3$. A shortest path is 000→100→110→111, requiring three edges.

Example 3: Ternary sphere
In F_3^3, the number of points at distance 2 from a fixed point is C(3,2)2^2=12. The formula illustrates why ternary geometry is richer than binary geometry at the same dimension.
Example 4: Quantum translation
For x=101 and a=110, the Pauli-X pattern X⊗X⊗I acts on |101⟩ and gives |011⟩, because 101+110=011 modulo 2.
Example 5: Binary image mismatch
For x=101101 and y=100001, the Hamming distance is 2. Their XOR mask is 001100, showing exactly the two positions at which the images differ.

XVII. COMPUTATIONAL AND ALGORITHMIC PERSPECTIVE

The framework is particularly suitable for computation because all binary operations can be implemented with bitwise XOR and bit tests. A vector in F_2^n can be stored as an n-bit word. Hamming weight and Hamming distance can then be evaluated efficiently by population-count operations on XOR results.

For q-ary systems, vectors can be stored as q-ary symbols or packed representations. Matrix transformations over finite fields can be used for coding, finite geometry and digital transformation tasks. The same coordinate labels can also be passed to quantum simulation software as computational-basis indices, while the quantum amplitudes remain complex numbers.

XVIII. PROPOSED RESEARCH PROBLEMS AND SCOPE

The following problems are proposed as directions for genuine further research. They are deliberately stated as open investigations rather than as established theorems.
1. Classification of graph-compatible near-ring multiplications on F_2^n for small n.
2. Determine necessary and sufficient conditions on a binary operation * for all left translations x↦a*x to preserve Hamming spheres.
3. Study q-ary modular cubical algebras on F_q^n and compare their automorphism groups.
4. Construct and classify digital transformations that are simultaneously affine over F_2 and graph automorphisms of Q_n.
5. Investigate coding-theoretic properties of subsets of F_q^n that are closed under a proposed near-ring multiplication.
6. Study Pauli-X/Weyl representations of modular translation structures and determine which finite algebraic symmetries have unitary realizations.
7. Define invariants measuring the compatibility between an algebraic operation and a graph metric.
8. Explore finite topological versions by placing compatible finite topologies on modular coordinate spaces and studying continuous algebraic transformations.
9. Develop computational algorithms for enumerating small modular cubical algebras and testing associativity, distributivity, identities, ideals and automorphisms.
10. Investigate whether graph-induced near-ring constructions can produce families not isomorphic to standard finite rings.

XIX. DISCUSSION

The main mathematical message is that modular coordinate systems provide a common finite language across several disciplines. In the binary case, F_2^n is simultaneously an n-dimensional vector space, a set of digital words, the vertex set of Q_n, a Hamming space and an indexing set for n-qubit computational basis states. These are different structures on the same underlying set, and the power of the framework comes from identifying exactly which maps preserve which structures.

The ternary case demonstrates that the construction is not restricted to binary mathematics. F_3^n provides a natural q-ary extension and connects to ternary coding and qutrit indexing. More generally, F_q^n gives a systematic finite coordinate model whenever q is prime. For composite q, Z_q^n remains a modular coordinate system, but vector-space statements over F_q must be replaced by ring-module statements or other appropriate algebraic structures.

The proposed modular cubical algebra programme is the point at which standard facts can be developed into

new research. A successful research paper in this direction should introduce an explicitly new operation or compatibility condition, establish nontrivial structural theorems, classify examples or counterexamples, and compare the resulting objects with known rings, near-rings, graph algebras and coding structures. Computational enumeration for orders 4, 8, 16 and 32 could provide a practical starting point.

XX. CONCLUSION

This paper has developed a broad unified framework beginning with the relationship between Z_2 and the vertices of the square and cube and extending through F_2^n, F_3^n and general q-ary coordinate spaces. The hypercube graph interpretation converts modular coordinate differences into graph distance, while Hamming geometry supplies the natural discrete metric for digital and coding applications. Boolean XOR becomes vector addition, error patterns become translations, and hypercube routing becomes Hamming-distance minimization.

The quantum-information extension gives an exact representation-level correspondence: binary translations x↦x+a are implemented on computational basis labels by tensor products of Pauli-X operators, while q-ary translations are represented by generalized shift operators. This bridge is mathematically useful precisely because it preserves the distinction between finite modular labels and complex quantum state spaces.

Finally, the proposed modular cubical algebra framework provides a path toward new research in finite algebra and near-rings. The most promising work lies not in rediscovering the classical hypercube correspondence, but in constructing and classifying algebraic operations compatible with graph, coding, digital and quantum-inspired structures. This provides a broad platform for future theoretical and computational investigations.

Appendix A. Basic Binary Tables

x	y	x+y in F_2	Hamming distance
00	00	00	0
00	01	01	1
00	10	10	1
00	11	11	2
01	10	11	2
01	11	10	1
10	11	01	1
11	11	00	0

Appendix B. Selected Cardinalities

n	|F_2^n|	|F_3^n|	|Z_4^n|
1	2	3	4
2	4	9	16
3	8	27	64
4	16	81	256
5	32	243	1024
6	64	729	4096
7	128	2187	16384
8	256	6561	65536

Appendix C. Terminological Caution

1. F_2^n is a vector space for every n, but for n>1 it is not a field under coordinatewise multiplication.
2. F_3^n is similarly a vector space, not a field for n>1.
3. Z_q is a field only when q is prime.
4. The hypercube vertex set is identified with F_2^n as a labelled set/vector space; Euclidean geometric operations should not be conflated with modular algebraic operations.
5. The quantum connection uses F_2^n or F_q^n as computational-basis labels; the physical state space remains a complex Hilbert space.
6. Claims of novelty should be restricted to newly constructed algebraic operations, classifications, invariants, proofs or experimentally verified applications.

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