SUSTAINABLE CONCRETE

An Empirical Model for the Relative Compressive Strength of Concrete Containing Rice Husk Ash, Palm Oil Fuel Ash and Sugarcane Bagasse Ash

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Abstract

Rice husk ash (RHA), palm oil fuel ash (POFA) and sugarcane bagasse ash (SCBA) are abundant agro-industrial residues that can partially replace Portland cement in concrete, but designers lack a simple, validated way to estimate how much strength a given replacement will cost. This study developed and tested a literature-derived empirical model for the strength ratio R, the compressive strength of an ash concrete divided by that of the plain cement concrete made with the same materials in the same laboratory. From a database of 736 records in 37 published experimental programmes, 574 ash-concrete records from 35 programmes were each paired with their own control and used for modelling. Nine candidate models were fitted by least squares with standard errors clustered by programme and tested by leave-one-study-out validation, in which every programme in turn was predicted by a model that had never seen it. The adopted model, $\ln R = -(5.46 - 3.94g)x^2 + 0.123x \ln(t/28)$, uses the replacement fraction x (ash mass divided by total binder mass), the curing age t in days and an indicator g that equals 1 when the source study reported milling the ash to standard fineness or beyond, as classified from the processing it described, and 0 otherwise. All three terms were statistically significant. Coding grinding in four ordered levels did not improve predictions, and neither did separate terms for the three ashes ($p = 0.74$) or loss on ignition. On programmes it had not seen, the model predicted R with a root-mean-square error of 0.166, against 0.205 for a constant median (bootstrap 95% interval of the reduction 0.007 to 0.067), and explained about one-third of the variance in R. The model indicates that unground ash can replace about 9% of the binder for a 5% loss of 28-day strength and that milling raises this to about 18%, or 23% if strength is assessed at 90 days. These are preliminary estimates for an average ash source; the scatter between sources remains large, so trial mixes are still required.

Keywords: agro-industrial ash; compressive strength; empirical model; grinding fineness; supplementary cementitious materials

1 Introduction

Portland cement production accounts for a large share of global anthropogenic carbon dioxide emissions, and most of that share comes from the calcination of limestone rather than from fuel, so it cannot be removed by cleaner kilns alone [1]. Partial replacement of clinker with supplementary cementitious materials is one of the few measures that can reduce it at scale. In industrialised economies this route rests on coal fly ash and ground granulated blast-furnace slag, which typically replace on the order of 10 to 50% of the cement in concrete [2]. Neither material is produced in useful quantities in most of sub-Saharan Africa and South-East Asia, where the demand for new concrete is growing fastest.

Those regions do produce large quantities of agro-industrial ash. Rice husk ash (RHA) remains after husk is burnt for energy at rice mills, palm oil fuel ash (POFA) after fibre and shells are burnt in palm oil mill boilers, and sugarcane bagasse ash (SCBA) after bagasse is burnt in sugar factory boilers. The three are considered together here because they share the features that govern their use: all are siliceous combustion residues of the same tropical agro-industries, all react pozzolanically with the calcium hydroxide released by cement hydration, and all are improved by the same processing, controlled burning and grinding. Their performance depends heavily on that processing: Hwang et al. [3] report that grinding an RHA from a mean particle size of 87 μm to 12 μm raised its 28-day strength activity index from 72% to 97% of the control mortar. Pooling them also gives more programmes than any one ash offers alone, and it allows the question of whether ash type matters, once processing is accounted for, to be tested rather than assumed.

Several kinds of tool exist for estimating the strength of concrete containing a pozzolan. The oldest is the efficiency factor, or k-value, introduced by Smith [4] for fly ash, which counts each kilogram of addition as k kilograms of cement in the water–cement ratio; Papadakis and Tsimas [5] later estimated k-values for several pozzolans from their composition. Wong and Razak [6] showed that k is not a constant: it falls as the replacement level rises and grows with age, which is the behaviour a model of ash concrete must reproduce. EN 206 [7] gives k-values only for fly ash (k = 0.4), silica fume and ground granulated blast-furnace slag; any other addition must first have its suitability established, so no k-value is codified for an agro-industrial ash. The strength activity index of ASTM C618 [8] requires a mortar with 20% of its cement replaced to reach 75% of the control strength at 7 or 28 days. The index screens an ash, but it says nothing about how strength changes with replacement level or age in concrete. Statistical syntheses of the literature are the next step. Wahab et al. [9] pooled published data on agricultural ashes in a meta-regression of 28-day strength on replacement level, fineness and age (R^2 = 0.71) and found fineness to be the strongest correlate of strength (r = 0.69). Their response, however, is absolute strength in MPa, which mixes the effect of the ash with that of each laboratory's cement and aggregates, and their model was assessed only on the data it was fitted to.

Machine learning offers a third route. Models trained on pooled RHA data [10]–[13] predict absolute strength from mix proportions with high accuracy, but they are generally evaluated on records withheld at random, which lets mixes from one programme fall on both sides of the split; when whole sources are withheld, such accuracy can fall to about zero [14]. Mirzaei et al. [15] took the step closest to this study, predicting the 28-day strength ratio of concretes containing 18 agricultural ashes with gradient-boosted trees and reaching a cross-validated R^2 of about 0.80, with folds built on ash type rather than on source. Their model differs from the present one in three respects: it is a black box rather than an equation, it requires the oxide composition and specific surface area of the ash, which are often unavailable at the design stage, and it predicts 28-day strength only.

The experimental literature on the three ashes is large [16]–[52]. Because each programme uses its own cement, aggregates, specimens and curing, the results are difficult to combine. Individual studies report an optimum replacement level for their own ash, often near 10%, but those optima vary widely and cannot be transferred to another ash source. The gap is therefore specific. No published model gives, in closed form, the strength of agro-ash concrete relative to its plain counterpart as a function of the quantities a designer controls, namely the replacement level, the processing of the ash and the age at which strength is assessed, and none has been tested on experimental programmes withheld from its fitting. The relative strength is the natural target for such a model: it cancels the laboratory-specific effects of cement, aggregates and specimen type, which are common to an ash mix

and its control, and it answers the designer's question directly, namely how much strength a given replacement costs.

The aim of this study was to develop and validate such a model. Its objectives were: (i) to compile, from published experimental programmes, a set of ash concretes each paired with a plain control made with the same materials; (ii) to formulate and fit candidate models of the strength ratio in terms of replacement level, grinding, curing age, ash type and loss on ignition, and to select among them by their accuracy on unseen programmes; and (iii) to derive from the adopted model the replacement levels that limit the strength loss to stated values, as a basis for preliminary mix design.

2 Methodology

2.1 Data collection and screening

Data were collected from published experimental programmes identified through Google Scholar and Consensus, with most searches run on 20 June 2026. The search combined the names and acronyms of the three ashes with terms for concrete, compressive strength and chemical composition. A programme was included if it reported the compressive strength of a binary concrete made with Portland cement (CEM I) and a single agro-industrial ash (RHA, POFA or SCBA) at a stated curing age, together with the 28-day strength of a plain cement control cast within the same programme. Ternary and quaternary binders, blends containing industrial by-products, concretes with aggregates other than normal-weight mineral aggregates, and steam-cured or field-cured specimens were excluded; where a programme tested several curing regimes, only the water-cured series was retained. Data were extracted from 41 programmes that met these criteria on screening, and four were removed after extraction: one duplicated an earlier study; in another, the replacement levels stated in the text and in the mix table could not be reconciled; a third used ash from date palm fronds rather than palm oil fuel ash; and the fourth made two-stage concrete with recycled coarse aggregate.

The resulting database contains 736 compressive-strength records from 37 programmes published between 2006 and 2026 [16]–[52], where a record is the mean strength of one mix at one age (Table 1). Of these, 140 records are plain controls and 596 contain ash. For every ash record, the mix proportions, the curing age, the oxide composition and loss on ignition (LOI) of the ash where reported, and a description of its processing were extracted. The 28-day control strengths range from 16.8 to 48.0 MPa, so the data describe normal-strength concrete. Figure 1 traces the records from extraction to the final modelling set: of the 596 ash records, 585 could be paired with a control at the same age, and the 11 records of the two programmes that did not describe how their ash was processed were then excluded, leaving 574 paired records from 35 programmes. All models in this paper were fitted and validated on these 574 records.

Table 1. Summary of the database by binder type.

Binder	Std.	Rec.	Repl. (%)	w/b	Age (d)	f (MPa)
Plain control	37	140	–	0.23–0.70	1–365	10.4–59.3
RHA	10	222	5–30	0.39–0.70	1–365	6.9–71.2
POFA	14	211	1–70	0.23–0.95	3–180	5.3–53.4
SCBA	13	163	5–50	0.40–0.67	7–180	7.5–55.9

Std., studies; Rec., records in the full database of 736 records; Repl., replacement by mass of total binder; w/b, water to binder ratio by mass, with the ash counted as binder; f, compressive strength as reported.

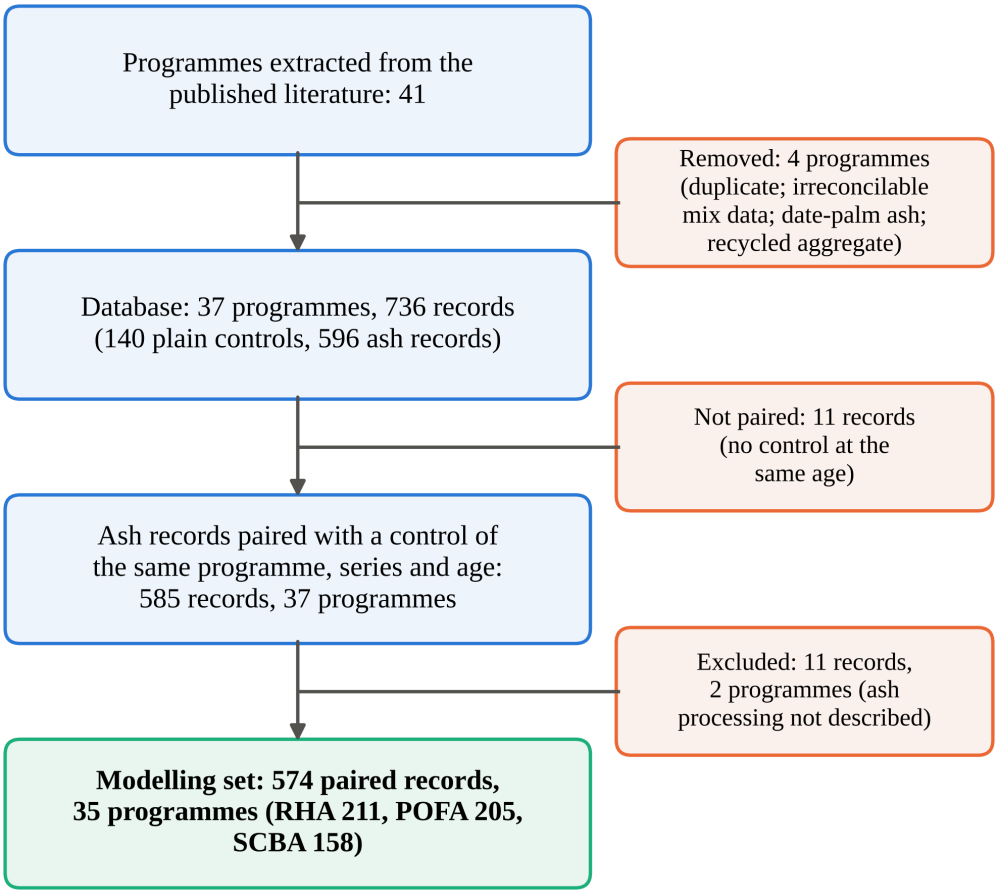


Figure 1. Selection of programmes and records, from extraction to the modelling set.

2.2 Response variable

The response modelled is the strength ratio

$$R = \frac{f_{\text{ash}}}{f_{\text{ctrl}}} \tag{1}$$

where f_{ash} is the compressive strength of an ash concrete and f_{ctrl} that of the plain control from the same programme, the same series of mixes and the same curing age. A series is a set of mixes that share a control, such as the mixes made at one water–binder ratio; where more than one control result was reported for a series and age, their mean was used. The ratio removes the effect of the cement, aggregates, workmanship and specimen geometry of each laboratory, which are common to an ash mix and its control, so that cube and cylinder results can be combined without conversion. R = 1 means that the ash concrete matches the plain concrete it replaces, and R = 0.95 means a 5% loss of strength.

2.3 Explanatory variables

Four quantities were considered. The replacement fraction is defined on the total binder mass,

$$x = \frac{m_{\text{ash}}}{m_c + m_{\text{ash}}} \tag{2}$$

where m_{ash} and m_c are the masses of ash and cement per cubic metre of concrete; x was computed from the reported mix proportions, so studies that described replacement as a percentage of cement are expressed on the same basis. The curing age t is in days and enters as ln(t/28), so that the age term vanishes at 28 days.

Fineness could not be used directly, because the programmes reported it in incompatible forms (Blaine surface area, median particle size, sieve residue or milling time) or not at all. The processing of each ash was therefore classified into one of four ordered levels from the description in its source paper: 1, used as received or only sieved; 2, coarse or short-duration milling; 3, standard ball milling to roughly cement fineness; and 4, prolonged or intensive milling to finer than cement. Where a paper reported a fineness measure, it was used to confirm the level, and the typical median particle size is roughly 10 to 20 μm at level 3 and below about 10 μm at level 4 [53]–[56]. The levels are therefore categories of reported processing, not measured fineness values for each ash. For the adopted model the levels were merged into a binary indicator g, equal to 1 for levels 3 and 4 (ground ash) and 0 for levels 1 and 2 (unground ash); the reason for this grouping is given in the section on model selection. Of the 574 modelling records, 273 used ash at levels 1 or 2 and 301 at levels 3 or 4. The ash type and, where reported, the LOI of the ash were tested as further variables.

2.4 Model formulation

The model was formulated in logarithms, so that its predictions of R are always positive and its errors are proportional rather than absolute. Its general form is

$$\ln R = \beta_1 x^2 + \beta_2 x \ln(t/28) + h(\text{grinding}) x^2 \quad (3)$$

Each term follows from a physical requirement or an observed trend. The model has no intercept because a concrete without ash is its own control: at x = 0 every term vanishes, so ln R = 0 and R = 1 exactly. The replacement enters as x^2 because the data show little loss of strength up to about 10% replacement and an accelerating loss beyond it (the section on the compiled data); a squared term reproduces this shape with a single coefficient, and a linear term was tested as an alternative. The age term x ln(t/28) is zero at 28 days by construction, follows the logarithmic gain of strength with age that underlies the time functions of Eurocode 2 [57], and is multiplied by x because the control gains strength too: it represents only the additional late gain of the ash concrete, which should grow with the amount of slowly reacting ash. Finally, the benefit of grinding grew with the replacement level within single programmes, so grinding enters as a modifier h of the curvature of the x^2 term rather than as a separate offset. Three codings of h were compared: equal steps between the four levels, a separate step for each level, and a single step between unground ash (levels 1 and 2) and ground ash (levels 3 and 4).

2.5 Estimation and validation

The coefficients of each candidate model were estimated by ordinary least squares, that is, by minimising the sum of squared differences between the observed and fitted values of ln R. Records from one programme share a cement, aggregates and procedures, so their errors are correlated and conventional standard errors would overstate the precision of the coefficients. Standard errors were therefore computed with the cluster-robust (sandwich) estimator, with the programme as the cluster [58], and the significance of each coefficient was judged from these.

The models were then tested by leave-one-study-out validation. Each of the 35 programmes was removed in turn, the model was refitted to the other 34, and the R of every record in the removed

programme was predicted. Every record therefore received one prediction from a model that had not seen its programme, which reproduces the situation of a designer applying the model to a new ash source and laboratory. From these withheld predictions $\hat{R}_i$ of the n = 574 observed ratios R_i, three measures were computed: the root-mean-square error, RMSE = $\sqrt{[\Sigma(\hat{R}_i - R_i)^2/n]}$; the mean absolute error, MAE = $\Sigma|\hat{R}_i - R_i|/n$; and the validation R^2, $1 - \Sigma(R_i - \hat{R}_i)^2/\Sigma(R_i - \bar{R})^2$, the share of the variance of R about its mean $\bar{R}$ that the withheld predictions explain. A baseline that predicts the median observed ratio (0.94) for every mix was used for comparison.

Whether one model was more accurate than another on new programmes was tested by a paired bootstrap. In each of 2,000 resamples, 35 programmes were drawn with replacement, the RMSE of both models was computed from the withheld predictions of the records in the drawn programmes, and the difference was recorded; the 2.5th and 97.5th percentiles of the 2,000 differences give its 95% interval, and an interval that excludes zero indicates a reliable improvement. The fitted R^2 is reported about the mean of ln R; because the models have no intercept, this is lower than the uncentred value printed by some software, and it describes fit, not predictive accuracy. All analyses were carried out in Python with the pandas, SciPy and statsmodels libraries.

3 Results and discussion

3.1 Behaviour of the compiled data

Figure 2 shows the 28-day strength ratios of the paired records against replacement level. Across the three ashes the median ratio is 1.03 at replacements up to 7.5% and 1.00 at 7.5 to 12.5%, so at these levels a typical ash concrete matches its control. Beyond 12.5% the ratio falls at an increasing rate, to 0.92 at 12.5 to 17.5%, 0.85 at 17.5 to 25%, 0.78 at 25 to 35% and 0.66 above 35%. This shape, flat at first and then falling ever more steeply, is the reason for the x^2 form of (3).

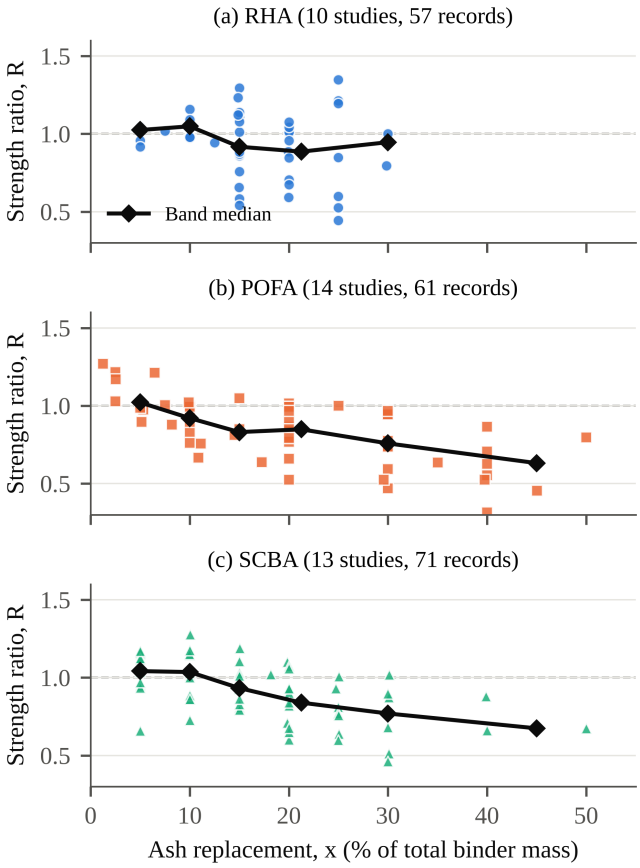


Figure 2. Ratio of the 28-day compressive strength of each ash concrete to that of its own plain control, against replacement level: (a) RHA, (b) POFA, (c) SCBA. The black line joins the medians of replacement bands with at least three records; the dashed line marks equal strength.

Two further patterns guided the choice of variables. Six programmes tested the same mix with the same ash at two or more grinding levels, giving 29 matched comparisons at 28 days, and in every one of them the more finely ground ash gave the higher ratio, by a median of 0.32. The gain grew with replacement level (Spearman $\rho = 0.45$, $p = 0.01$): in Tangchirapat et al. [49], grinding raised the ratio from 0.76 to 0.99 at 10% POFA but from 0.32 to 0.87 at 40%. And of 72 ash mixes tested at both 28 and 90 days, 55 gained on their control between those ages, the median ratio rising from 0.92 to 0.98 (Wilcoxon $p < 0.001$). Grinding and curing age therefore both act on the ratio, and both act more strongly at higher replacement.

3.2 Model selection

Table 2 compares the candidate models. Two kinds of measure are reported, and they answer different questions: the fitted R^2 describes how closely a model reproduces the data it was fitted to, whereas the validation errors describe how well it predicts programmes it has not seen. Selection was based on the validation errors. Replacement level alone (M1) reduces the validation RMSE from 0.205 to 0.181, and curing age (M2) changes little. Grinding coded in four equal steps (M3) lowers the RMSE to 0.173. Giving each level its own step (M4) showed why the equal-step coding is not adequate: the step from level 1 to level 2 was 2.11 ($p = 0.18$), the step from level 2 to level 3 was 2.14 ($p = 0.003$), and the step from level 3 to level 4 was −0.42 ($p = 0.66$). Across programmes, then, the data resolve one contrast only, between ash that was milled to standard fineness or beyond and ash that was not. A single step between those two groups (M5) gave the lowest validation errors of all the

models, with an RMSE of 0.166, an MAE of 0.131 and a validation R^2 of 0.33, with only three coefficients, all significant, and it was adopted.

Table 2. Candidate models and their accuracy in leave-one-study-out validation.

Model	k	R^2 (fit)	RMSE	MAE	R^2 (val.)
Baseline: median ratio	0	–	0.205	0.159	−0.03
M1: replacement only	1	0.21	0.181	0.137	0.20
M2: M1 + curing age	2	0.24	0.180	0.137	0.21
M3: M2 + grinding, 4 equal steps	3	0.40	0.173	0.133	0.27
M4: M2 + grinding, step per level	5	0.45	0.172	0.135	0.28
M5: M2 + ground or not (adopted)	3	0.44	0.166	0.131	0.33
M6: M5 + linear term in x	4	0.44	0.171	0.134	0.29
M7: M5, curvature per ash	5	0.45	0.179	0.138	0.22
M5 on LOI subset (391)	3	0.44	0.179	0.143	0.32
M8: M5 + LOI (391)	4	0.47	0.180	0.144	0.31

k, number of coefficients; R^2 (fit), centred R^2 of ln R with all records; RMSE, MAE and R^2 (val.) are for R in leave-one-study-out validation (574 records and 35 programmes, except the last two rows).

Three extensions of M5 were rejected. A linear term in x (M6) took a negative value (−0.15, p = 0.51) and worsened prediction, so the pooled data give no support for a gain in strength at low replacement; the medians slightly above 1 in Figure 2 are within the scatter between programmes. Giving each ash its own curvature (M7) worsened the validation RMSE to 0.179, and the three curvatures did not differ significantly (Wald test, p = 0.74). Adding LOI (M8), on the 391 records that report it, left the validation RMSE practically unchanged (0.179 to 0.180). All three extensions raised the fitted R^2 slightly, as any added term must, but none improved prediction of unseen programmes: the extra coefficients fitted variation peculiar to the programmes in the data rather than a transferable effect.

The gains between the best models are small, and the bootstrap shows which are reliable. M5 improved on the median baseline by 0.039 in RMSE (95% interval 0.007 to 0.067), and was more accurate for 22 of the 35 programmes. Its improvement over M2, the model without grinding, was 0.014 (95% interval −0.011 to 0.032), which is not statistically significant: grinding adds little to the prediction of a new programme taken as a whole. The grinding term was nonetheless retained, for three reasons. Its coefficient is significant (Table 3), its sign agrees with all 29 within-programme comparisons, and it changes the replacement a designer can use by a factor of about two (see Implications for mix design).

3.3 The adopted model

The fitted coefficients of M5 are given in Table 3. All three are statistically significant with standard errors clustered by programme, and the signs are as expected: replacement lowers the ratio, grinding reduces that penalty, and the ratio rises with age beyond 28 days.

Table 3. Coefficients of the adopted model (M5).

Coefficient	Estimate	Clustered SE	95% CI	p
β_1 (x^2)	−5.459	1.261	−7.93 to −2.99	< 0.001
β_2 (x ln(t/28))	0.123	0.052	0.02 to 0.22	0.017
β_3 (g x^2)	3.935	1.334	1.32 to 6.55	0.003

g = 1 for ash milled to grinding level 3 or 4, 0 otherwise. Fitted to 574 paired records from 35 programmes; residual standard deviation of ln R 0.19. The estimates of β_1 and β_3 are strongly correlated (r = −0.96), so their sum, the curvature for ground ash, is better determined than either alone. Across the 35 leave-one-study-out refits, β_1 ranged from −6.24 to −4.14, β_2 from 0.106 to 0.153 and β_3 from 2.65 to 4.71.

Written out, the model is

$$R = \exp\left[-(5.46 - 3.94g)x^2 + 0.123x\ln(t/28)\right] \quad (4)$$

where R is the predicted ratio of the compressive strength of the ash concrete to that of a plain CEM I concrete made with the same materials and tested at the same age; x is the ash replacement as a fraction of the total binder mass, Eq. (2), valid for 0 ≤ x ≤ 0.40; t is the curing age in days, within the 7 to 180 days of most of the data; and g = 1 for ash reported as milled to standard fineness or beyond (levels 3–4) and 0 otherwise. The predicted strength of the ash concrete follows as

$$f_{\text{ash}} = R \times f_{\text{ctrl}} \quad (5)$$

where f_{ctrl} is the strength of the plain concrete at the same age. For example, for a plain concrete of 30 MPa at 28 days, 20% of ground ash gives R = exp(−1.52 × 0.04) = 0.94 and a predicted strength of about 28.2 MPa.

Milling to standard fineness removes about 70% of the curvature of the unground-ash curve (3.94 of 5.46), so that ground ash loses strength far more slowly with replacement. At 20% replacement and 28 days, (4) predicts R = 0.80 for unground ash and 0.94 for ground ash. The age term is smaller: at 20% replacement, the ratio at 90 days is about 3% higher than at 28 days. The coefficients were stable across the leave-one-study-out refits, so no single programme determines them.

The two-group coding is a simplification that errs on the safe side. Within single programmes, grinding from level 1 to level 2 raised the ratio by 0.09 to 0.32 in El-said et al. [21], and grinding from level 3 to level 4 raised it by 0.06 to 0.20 in Tangchirapat and Jaturapitakkul [48]. Across programmes these steps are hidden by other differences between laboratories, so (4) treats level 2 ash as unground and level 4 ash as ground, and it is likely to underestimate both.

The rejection of ash-specific terms deserves comment. The three ashes differ in chemistry: RHA is the most siliceous, and POFA the most variable, with only 6 of its 14 reported analyses meeting the 70% minimum for $SiO_2 + Al_2O_3 + Fe_2O_3$ that ASTM C618 sets for natural pozzolans [8], against 9 of 10 for RHA and 11 of 15 for SCBA. Their median ratios also differ at a given replacement level. Once grinding and curing age are taken into account, however, the remaining differences between the ashes are smaller than the differences between programmes using the same ash. This result should be read narrowly. It does not show that the three ashes perform alike; it shows that, with 10 to 13 programmes per ash, an effect of ash type could not be separated from the effect of the laboratory, whereas the effect of grinding was large enough to be detected.

3.4 Validation on unseen studies

Table 4 and Figure 3 report the accuracy of (4) on programmes withheld from fitting. Overall, the RMSE is 0.166 and the MAE 0.131, against 0.205 and 0.159 for the median baseline, a reduction of about 19%. Half of all predictions fell within ±0.10 of the observed ratio and 77% within ±0.20, and the mean bias was small (−0.016). Accuracy was similar for unground and ground ash. The model was most accurate for POFA and SCBA, for which the predicted and observed ratios were well correlated (Spearman ρ = 0.75 and 0.63), and least accurate for RHA (ρ = 0.33). More than half of the RHA error comes from a single programme [52], whose ratios range from 0.31 to 1.49 across its ashes, mixes and ages, and two others [38], [44] obtained ratios about 0.18 higher than predicted.

Table 4. Accuracy of the adopted model in leave-one-study-out validation.

Subset	Records / studies	RMSE	MAE	±0.10	Base. RMSE
All records	574 / 35	0.166	0.131	49%	0.205
RHA	211 / 10	0.184	0.142	47%	0.199
POFA	205 / 13	0.151	0.122	52%	0.223
SCBA	158 / 12	0.160	0.126	48%	0.189
Unground ash	273 / 22	0.166	0.134	45%	0.222
Ground ash	301 / 17	0.167	0.128	52%	0.189
28 days	181 / 35	0.167	0.131	50%	0.199
90 days or more	120 / 13	0.179	0.136	48%	0.202

errors are in units of the strength ratio R. ±0.10, share of predictions within ±0.10 of the observed ratio. Base., baseline: the median observed ratio applied to every record. Some programmes tested both unground and ground ash.

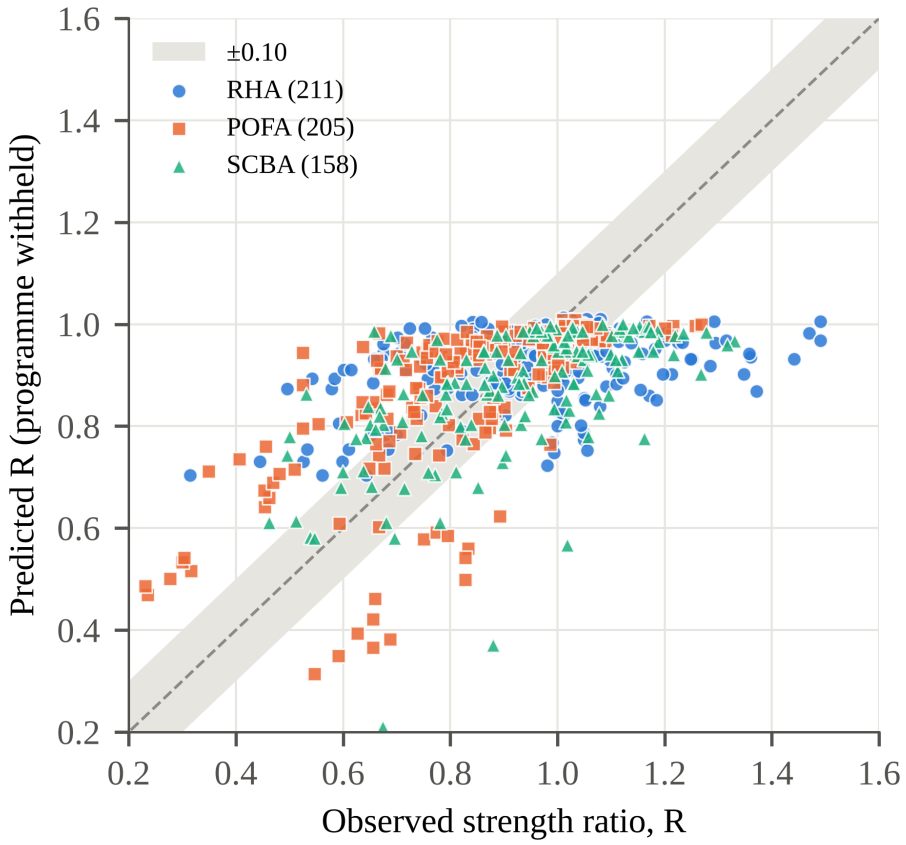


Figure 3. Predicted against observed strength ratio for every record, each predicted by a model fitted without the programme that reported it. The shaded band marks ±0.10.

Figure 3 also shows the limits of the model. Its predictions span a narrower range than the observations: very weak and very strong ash concretes are pulled towards the middle, as expected of a model that describes the average response across many ash sources. The validation R^2 of 0.33 means that the model explains about one-third of the variation of R between records; the remaining two-thirds is substantial and reflects differences between ashes and laboratories that replacement level, grinding and age cannot capture. In practical terms, 80% of the observed ratios lay between 0.76 and 1.27 times the predicted value. The model is therefore useful for estimating the typical strength penalty of a replacement and for comparing options, not for predicting the strength of a particular mix with precision.

3.5 Implications for mix design

Table 5 inverts (4) to give the largest replacement at which the predicted ratio stays at or above 0.95 or 0.90, that is, a strength loss of at most 5% or 10% relative to the plain concrete, and Figure 4 plots the corresponding curves. Two values are given for each case. The central value uses the fitted coefficients and applies to an average ash source. The lower value is the replacement at which the lower one-sided 95% confidence limit of the mean ratio reaches the target; it allows for the uncertainty in the coefficients and is the more cautious choice.

Table 5. Largest replacement x (% of total binder mass, rounded down) for a target strength ratio, from Equation (4).

Ash and age	R ≥ 0.95, central	R ≥ 0.95, lower	R ≥ 0.90, central	R ≥ 0.90, lower
Unground, 28 d	9	8	13	11
Unground, 90 d	11	9	15	12
Ground, 28 d	18	15	26	22
Ground, 90 d	23	17	31	24

unground, grinding levels 1–2; ground, levels 3–4. Central, from the fitted coefficients; lower, from the lower one-sided 95% confidence limit of the mean predicted ratio. Values are preliminary estimates for an average ash source; an individual ash source can depart further from the mean (the section on validation).

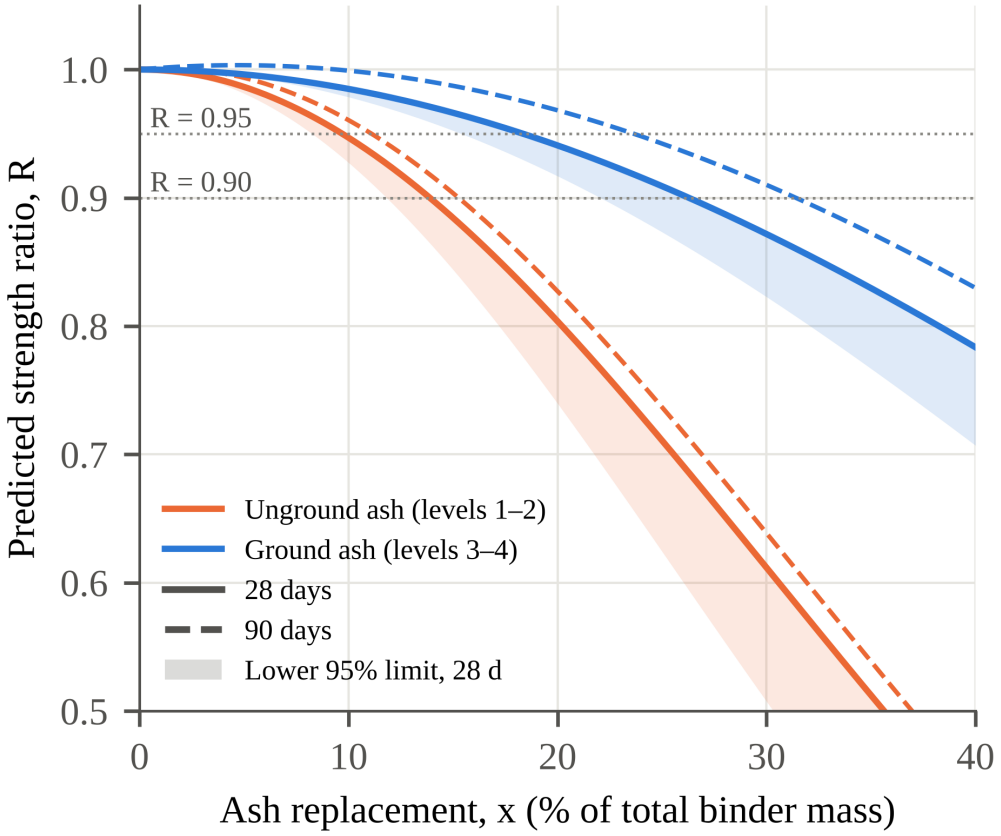


Figure 4. Strength ratio predicted by equation (4) for unground and ground ash at 28 days (solid) and 90 days (dashed). The shaded bands extend to the lower one-sided 95% confidence limit of the mean at 28 days.

For unground ash, a replacement of about 9% costs no more than 5% of the 28-day strength, and about 13% costs no more than 10%. Individual studies commonly report their highest strength near 10% replacement. Equation (4) has no such maximum, because the pooled data showed no gain in strength at low replacement (M6), so 10% is better read as the level up to which the loss is small than as an optimum. Grinding changes the picture substantially. Milling to standard fineness raises the 5% limit to about 18% at 28 days, and the predicted loss at 30% replacement is 13%, against 39% for unground ash. Assessing strength at 90 days adds a further 2 to 5 percentage points of replacement. Since the three ashes could not be distinguished once grinding was accounted for, these limits are offered for all three, subject to the larger uncertainty found for RHA.

An engineer would use the model in four steps: classify the ash as ground or unground from its supplier's processing; read a trial replacement from Table 5, or compute R from (4) for any x up to 0.40; estimate the strength of the ash concrete from (5), using the strength of the plain concrete it replaces; and confirm the result with a trial mix using the actual ash. The limits in Table 5 are preliminary estimates for proportioning, not design values for a particular ash, and they concern compressive strength only; workability, which unburnt carbon and fine grinding both affect, and durability must be checked separately before a mix is adopted.

4 Limitations

The model inherits the limitations of the published data. Fineness was reported inconsistently, or not at all, so grinding enters as a classification of the processing each paper described rather than as a measured property, and an independent recoding would strengthen confidence in β_3. Ground ash at level 4 comes from RHA and POFA programmes only, and no SCBA programme milled its ash that far. The three ashes are unequally represented (10 to 13 programmes each), and programmes contribute unequal numbers of records; although the standard errors and the validation both respect the grouping by programme, prolific programmes still weigh more in the fit. Variables that differ between programmes, such as ash chemistry, combustion conditions, curing practice and the strength of the base concrete, could not be separated from one another, which is why LOI and ash type did not improve the predictions and why two-thirds of the variance of R remains unexplained. The evidence beyond 28 days comes from 13 programmes, and only a few extend beyond 180 days. Data above 40% replacement are sparse (13 records, 12 of them POFA), and the model underpredicted them by 0.14 on average, so it should not be used beyond 40% replacement, nor outside normal-strength concrete (28-day control strength of 16.8 to 48.0 MPa). Finally, the model predicts compressive strength only.

5 Conclusion

A literature-derived empirical model of the strength of concrete containing RHA, POFA or SCBA relative to its plain counterpart was developed from 574 paired records in 35 published programmes and tested by leave-one-study-out validation.

1. The principal contribution is a closed-form model, $R = \exp[-(5.46 - 3.94g)x^2 + 0.123x \ln(t/28)]$, in which all three terms are statistically significant; the predicted strength of an ash concrete is R times the strength of the plain concrete.
2. On programmes it had not seen, the model predicted R with an RMSE of 0.166, about 19% lower than a constant median, and explained about one-third of the variance in R; the remaining variation between ash sources is substantial.
3. Grinding was the only ash property whose effect the pooled data resolved; separate terms for ash type and LOI did not improve prediction of unseen programmes.
4. As preliminary estimates, unground ash can replace about 9% of the binder for a 5% loss of 28-day strength and ground ash about 18%, rising by 2 to 5 percentage points if strength is assessed at 90 days.

The model is a tool for preliminary proportioning and for comparing options; it does not replace trial mixes with the actual ash. Future studies would make such models more precise by reporting the

fineness of the ash in a standard form, such as the median particle size, and by testing the same ash at several fineness levels and ages.

Data availability

The database was compiled as part of the first author's MSc research at the University of Ibadan. The compiled database and the analysis code are available from the corresponding author on reasonable request.

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